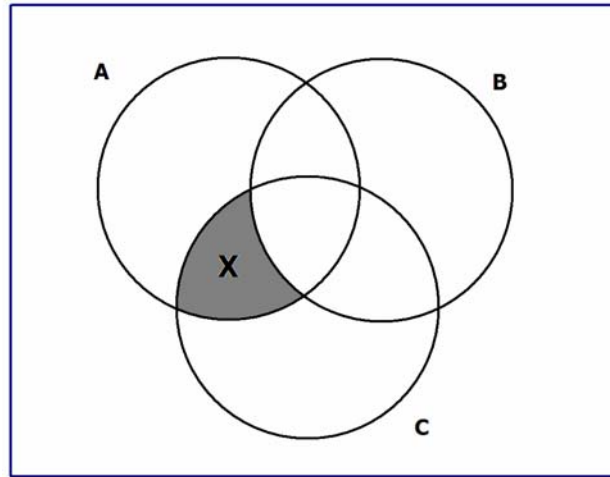


1.)



$$X = A \cap C - (A \cap B \cap C)$$

2.)

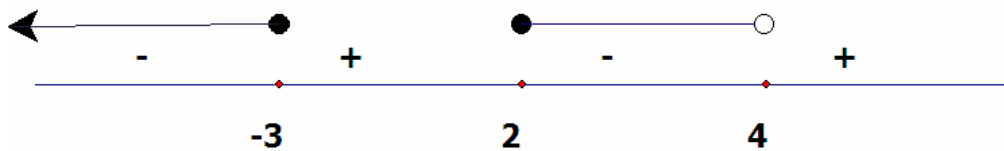
1.  $A \neq B \rightarrow A - B \neq \emptyset$  (incorrect)
2.  $A \cup C = B \cup C \rightarrow A = B$  (incorrect)
3.  $A \cup B = A \cap C \rightarrow A = B$  (correct)
4.  $A - B = \emptyset \rightarrow A = B$  (incorrect)

3.)

1.  $A = \frac{1}{2} \rightarrow a^2 = a$  (incorrect)
2.  $(a-b)^2 \geq 0 \rightarrow a^2 + b^2 \geq 2ab$  (correct)
3.  $\sqrt{(a-b)^2} = |a-b|$  (incorrect)
4.  $\sqrt{a^2} = |a|$  (incorrect)

4.)

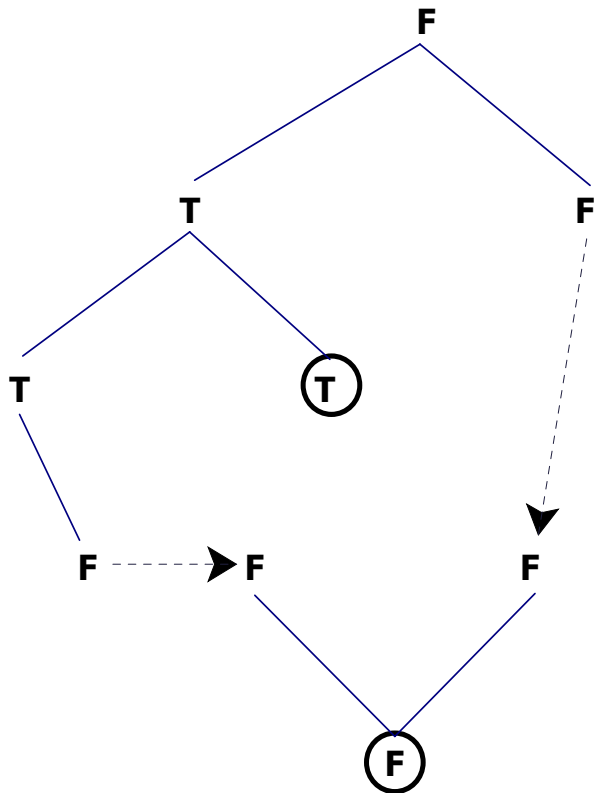
$$\frac{(x-2)(x+3)}{(x-4)} \leq 0$$



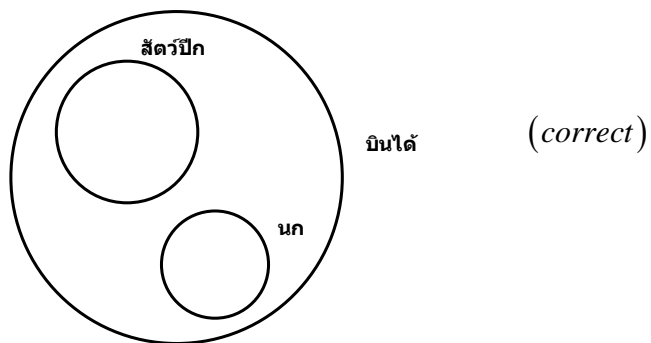
เซตคำตอบคือ  $(-\infty, -3] \cup [2, 4)$

5.)

6.)



7.)



8.)

9.)

$$\begin{aligned} 17^3 \cdot 17^x &= 17^{5x} \\ 5x &= x + 3 \\ 4x &= 3 \\ \therefore x &= \frac{3}{4} \end{aligned}$$

10.)

$$1. \log_{a^b} M = \frac{1}{b} \log_a M \quad (\text{incorrect})$$

$$2. y = \log_a M \rightarrow a^y = M$$

$$\rightarrow a^{\log_a M} = M \quad (\text{correct})$$

$$3. \log_N M = \frac{\log_M M}{\log_M N} = \frac{1}{\log_M N} \quad (\text{correct})$$

4.

11.)

$$2x^3 + 4x^2 - 6x - 12 = 2x^2(x+2) - 6(x-2)$$

$$= (2x^2 - 6)(x+2)$$

$$= 2(x - \sqrt{3})(x + \sqrt{3})(x+2)$$

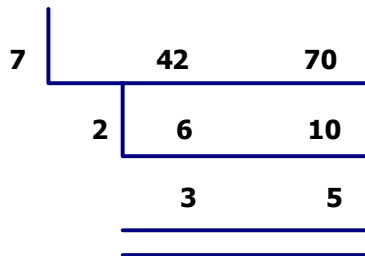
12.)

$$\text{п. } \sqrt{18}\sqrt{75} - \sqrt{24} = \sqrt{9 \cdot 25 \cdot 2 \cdot 32} \sqrt{6}$$

$$= 15\sqrt{6} - 2\sqrt{6}$$

$$= 13\sqrt{6} \quad (\text{incorrect})$$

у.

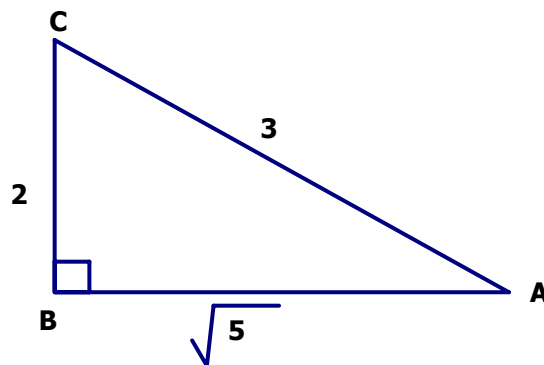


$$42 = 7 \times 2 \times 3$$

$$70 = 7 \times 2 \times 5$$

$$\therefore (42, 70) = 14$$

13.)



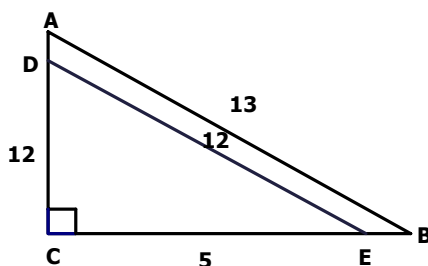
$$\operatorname{cosec}(B-C) = \operatorname{cosec} A = \frac{3}{2}$$

$$\cot A + \cot B + \cot C = \frac{\sqrt{5}}{2} + 0 + \frac{2}{\sqrt{5}}$$

$$= \frac{9}{2\sqrt{5}}$$

$$= \frac{9\sqrt{5}}{10}$$

14.)



$$\tan B = 2.4 = \frac{24}{10} = \frac{5}{2}$$

$$\triangle ABC \sim \triangle DEC$$

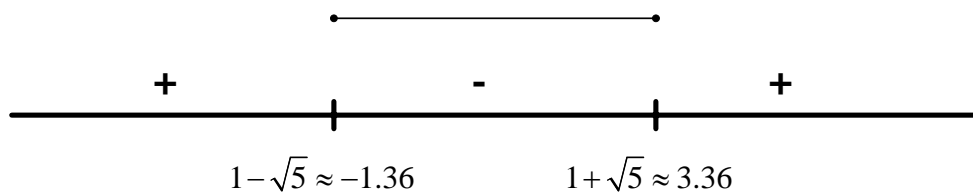
$$\frac{|CD|}{|AC|} = \frac{|DE|}{|AB|} \rightarrow |CD| = \frac{12 \cdot 12}{13}$$

15.)

$$|AD| = |AC| - |CD| = 12 - \frac{12 \cdot 12}{13} = 12 \left(1 - \frac{12}{13}\right) = \frac{12}{13}$$

$$x^2 - 2x - 4 \leq 0$$

$$x = \frac{2 \pm \sqrt{4+16}}{2} = 1 \pm \sqrt{5} \quad ; \quad (\sqrt{5} \approx 2.36)$$



$$A = \{-1, 0, 1, 2, 3\}$$

16.)  $(f \circ g \circ h)(x) = 4^{3(x^2-3x-3)} = \frac{1}{4} \cdot 4^x = 4^{x-1}$

$$3x^2 - 9x - 9 = x - 1$$

$$3x^2 - 10x - 8 = 0$$

$$(3x+2)(x-4) = 0$$

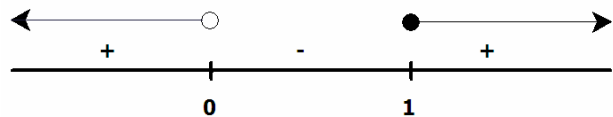
$$\therefore x = -\frac{2}{3}, 4$$

17.)

$$y = \frac{1}{1-|x|} \rightarrow y - |x|y = 1$$

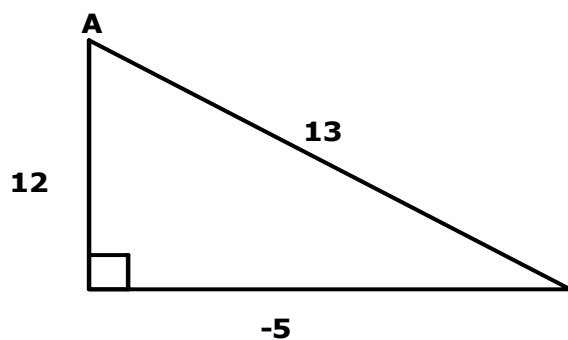
$$\therefore |x| = \frac{y-1}{y} \rightarrow |x| \geq 0, \frac{y-1}{y} \geq 0$$

$$\therefore y(y-1) \geq 0$$



เซตคำตอบคือ  
 $(-\infty, 0) \cup [1, \infty)$

18.)

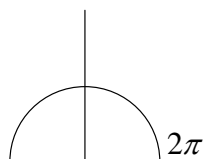


$$A = \arcsin\left(-\frac{5}{13}\right)$$

$$\text{ให้ } \sin A = -\frac{5}{13}$$

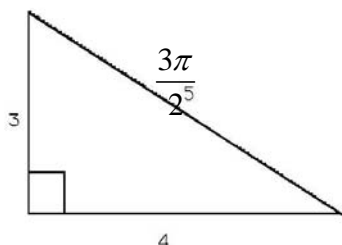
$$\therefore \cos A = \frac{12}{13}$$

19.) 1.



$$\sin \theta \leq \cos \theta \quad (\text{incorrect})$$

2.



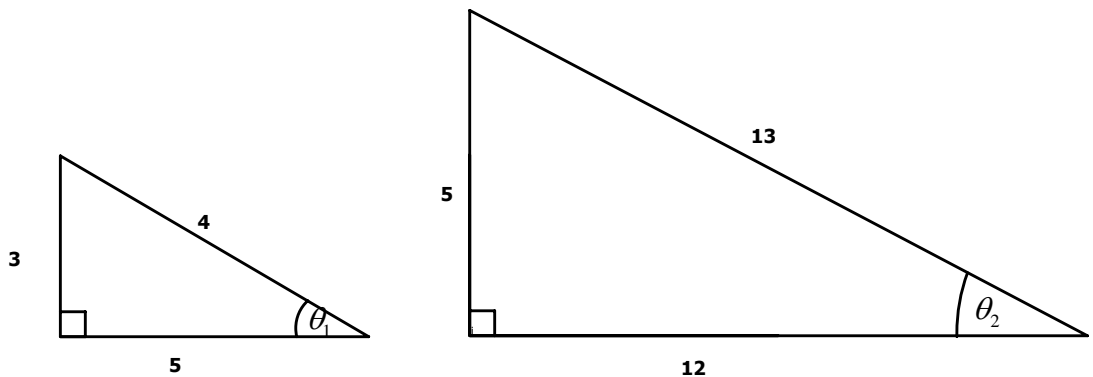
$$\cos \frac{\pi}{4} + \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \neq \sin \frac{\pi}{2} = 1 \quad (\text{incorrect})$$

3.

$$\sec \theta + \csc \theta = \frac{5}{4} + \frac{5}{3} = \frac{15+20}{12} = \frac{35}{12} \quad (\text{correct})$$

$$4. \quad \cos \theta = \frac{14}{25} < 1 \quad \therefore \quad \theta \text{ อยู่ใน } Q_1 \text{ หรือ } Q_4 \quad (\text{incorrect})$$

20.)



$$\begin{aligned} \sin(\theta_1 + \theta_2) &= \sin \theta_1 \cos \theta_2 + \cos \theta_1 \sin \theta_2 \\ &= \left(\frac{3}{5} \times \frac{5}{13}\right) + \left(\frac{4}{5} \times \frac{12}{13}\right) = \frac{15+48}{65} = \frac{63}{65} \end{aligned}$$

21.)

$$|\vec{u}| = 3$$

$$|\vec{u} + \vec{v}| = 4 \rightarrow |\vec{u}|^2 + |\vec{v}|^2 + 2|\vec{u}||\vec{v}|\cos \theta = 16$$

$$9 + |\vec{v}|^2 + 2 \cdot 3 \cdot |\vec{v}| \left(-\frac{3}{5}\right) = 16$$

$$\text{ให้ } |\vec{v}| = x \quad ; \quad 5x^2 - 18x - 35 = 0$$

$$(5x+7)(x-5) = 0$$

$$x = -\frac{7}{5}, 5 \quad \therefore |\vec{v}| = 5$$

22.)

$$\vec{u} = a\vec{i} + b\vec{j} + c\vec{k} \text{ โคลิโนและทิศทางของ } \vec{u} \text{ คือ } \left( \frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}} \right)$$

$$\text{และ } \left( \frac{a}{\sqrt{a^2+b^2+c^2}} \right)^2 + \left( \frac{b}{\sqrt{a^2+b^2+c^2}} \right)^2 + \left( \frac{c}{\sqrt{a^2+b^2+c^2}} \right)^2 = 1$$

$$\text{พิจารณา ข้อ 4 } \left( \frac{2}{\sqrt{17}} \right)^2 + \left( -\frac{3}{\sqrt{17}} \right)^2 + \left( \frac{4}{\sqrt{17}} \right)^2 \neq 1$$

23.)

$$|\vec{u}| = 3, |\vec{v}| = 4, \theta = 30^\circ$$

$$1. \quad \vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos \theta = 3 \cdot 4 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3} \quad (\text{correct})$$

$$2. \quad |\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin \theta = 3 \cdot 4 \cdot \frac{1}{2} = 6 \quad (\text{correct})$$

24.)

$$\det(A) = 2, \det(B) = 32$$

$$\det(B) = 16 \det(C)$$

$$\therefore \det(C) = \frac{32}{16} = 2$$

$$\det(A^{-1}C) = \frac{\det(C)}{\det(A)} = \frac{2}{2} = 1$$

25.)

จาก  $A^{-1} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$$

จะได้  $(A^{-1})^{-1} = A = \frac{1}{10} \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$

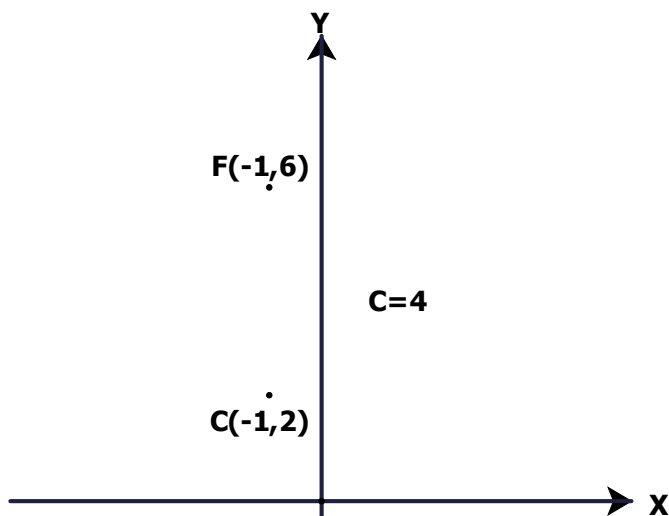
นั่นคือ  $a_{12} = \frac{4}{10} = \frac{2}{5}$

26.)

ข้อ 1,2  $\left(-\frac{46}{15}, \frac{14}{25}\right)$  ไม่อยู่บน  $3x - 5y = 12$

ข้อ 4  $\left(\frac{46}{25}, -\frac{14}{15}\right)$  ไม่อยู่บน  $3x - 5y = 12$

27.)



$$\frac{(x+1)^2}{b^2} + \frac{(y-1)^2}{a^2} = 1$$

$$(-3, 2); \frac{4}{b^2} + 0 = 1$$

$$\therefore b^2 = 4$$

$$a^2 = b^2 + c^2 = 4 + 16 = 20$$

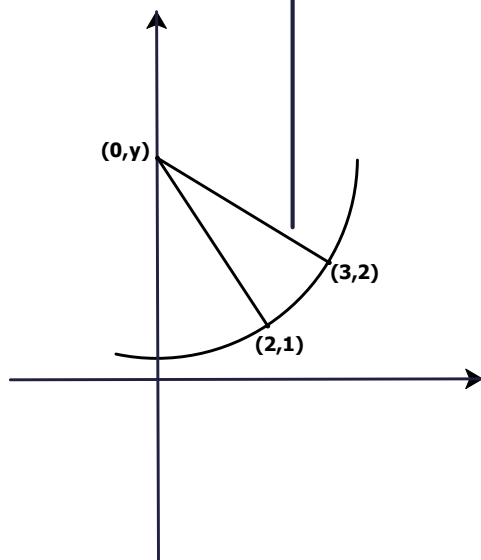
ผ่านจุด  $\therefore \frac{(y-2)^2}{20} + \frac{(x+1)^2}{4} = 1$

$$y^2 - 4y + 4 + 5x^2 + 10x + 5 = 20$$

$$5x^2 + y^2 + 10x - 4y - 11 = 0$$

$$\therefore d = -1, k = -11$$

28.)

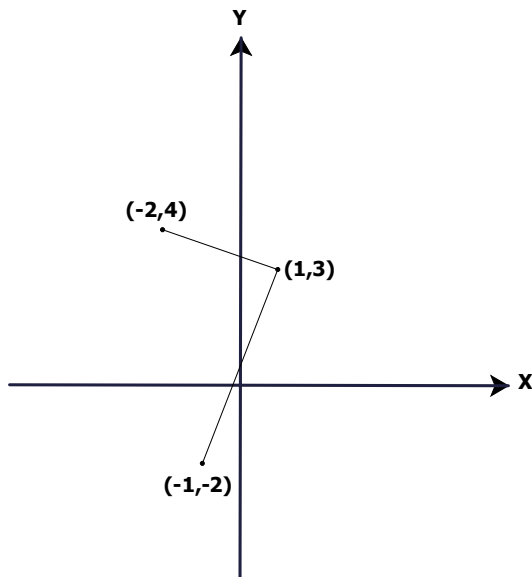


$$2^2 + (y-1)^2 = 3^2 = 3^2 + (y-2)^2$$

$$4 + y^2 - 2y + 1 = y^2 - 4y + 4 + 9$$

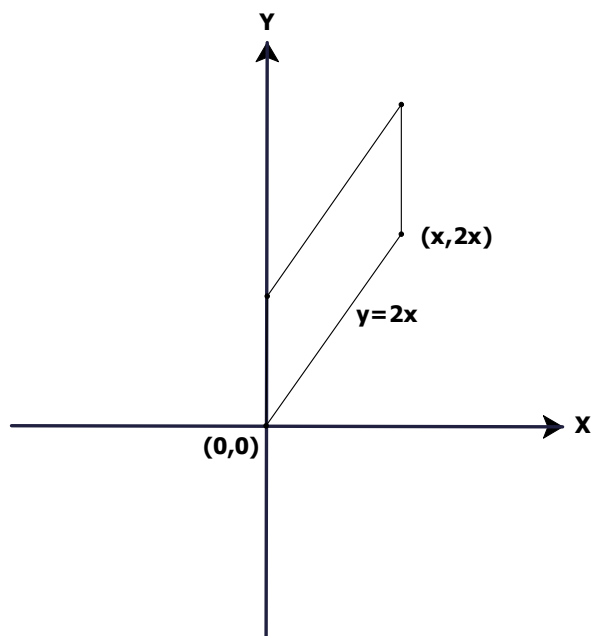
$$2y = 8 \rightarrow y = 4$$

29.)



1. ไม่ผ่านจุด  $(-1, -2)$
2. ไม่ผ่านจุด  $(-1, -2)$
3. ผ่านจุด  $(-1, -2)$
4. ไม่ผ่านจุด  $(-1, -2)$

30.)



$$\begin{aligned}
 x^2 + 5x^2 &= 5^2 \\
 x^2 &= 5 \rightarrow x = \sqrt{5} \\
 \therefore y &= 2\sqrt{5} \\
 x &= \sqrt{5} \\
 \therefore y &= 2\sqrt{5} + 5
 \end{aligned}$$

31.)

$$\begin{aligned}
 x^2 + y^2 + 4x - 6y - 12 &= 0 \\
 (x+2)^2 + (y-3)^2 &= 12 + 4 + 9 = 25 \\
 \text{ดังนั้นจุดศูนย์กลางของวงกลม คือ } &(-2, 3)
 \end{aligned}$$

$$\text{จากโจทย์ } \frac{d}{dx}(x^2 + y^2 + 4x - 6y - 12) = \frac{d}{dx}(0)$$

$$2x + 2yy' + 4 - 6y' = 0$$

$$\therefore 2x + 2yy' + 4 - 6y' = 0$$

$$8y' = -6$$

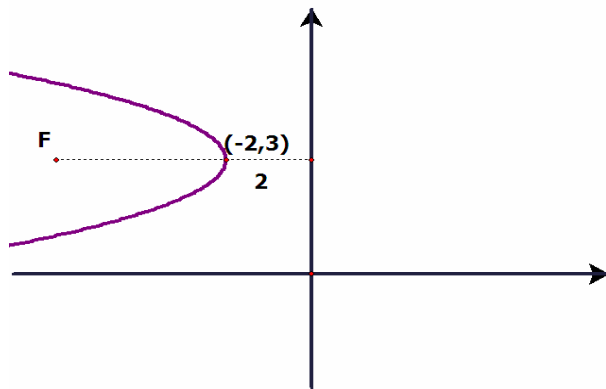
$$y' = -\frac{6}{8} = -\frac{3}{4}$$

$$\text{สมการเส้นตรง คือ } y - 3 = -\frac{3}{4}(x + 2)$$

$$4y - 12 = -3x - 6$$

$$3x + 4y - 6 = 0$$

32.)



$$x^2 + y^2 + 4x - 6y - 38 = 0$$

$$(x + 2)^2 + (y - 3)^2 = 38 + 4 + 9$$

$$(y - 3)^2 = 4(-2)(x + 2)$$

$$y^2 - 6y + 9 = -8x - 16$$

$$y^2 + 8x - 6y + 25 = 0$$

33.)  $1, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 5, \dots [n(1) = 1, n(2) = 2, n(3) = 4, n(4) = 8, \dots]$

เนื่องจาก  $1 + 2 + 4 + 8 + \dots$  เป็นเรขาคณิต

$$S_n = \frac{a_1(r^n - 1)}{r - 1} \rightarrow \frac{1(2^n - 1)}{2 - 1} \geq 2000$$

$$2^n - 1 \geq 2000$$

$$2^n \geq 2001$$

$$\therefore 2^n = 2048$$

$$2^{10} = 1024$$

$$\therefore a_{2000} = 11$$

34.)  $4, a, 9, b, c$  เป็น 8 พจน์ของลำดับเรขาคณิต  $a > 0$

$$\therefore a = \sqrt{4 \cdot 9} \rightarrow a = 6$$

$$r = \frac{6}{4} = \frac{3}{2} \rightarrow r = \frac{3}{2}$$

$$b = 9 \cdot \frac{3}{2} = \frac{27}{2}$$

$$c = 9 \cdot \frac{3}{2} \cdot \frac{3}{2} = \frac{81}{4}$$

$$a + 8c = 6 + 8\left(\frac{81}{4}\right) = 6 + 162 = 168$$

35.)  $101, 111, 121, 131, \dots, 191$



•  
•  
•

$$909, 919, 929, 939, \dots, 999$$

$$n(S) = 999 - 100 + 1 = 900$$

$$n(E) = 10 \cdot 9 = 90$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{90}{900} = \frac{1}{10}$$

$$36.) \quad n(S) = 6 \cdot 6 \cdot 6 = 216$$

$E$ : ผลรวมแต้มมากกว่าหรือเท่ากับ 4 จะได้  $n(E) = 216 - 1 = 215$  (1 คือ ออก 1 ทั้งสามหน้า)

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{215}{216}$$

$$37.) \quad Z^3 = \frac{1}{2} + \frac{\sqrt{3}}{2}i = \cos 60^\circ + i \sin 60^\circ$$

$$Z^{30} = \cos 10 \frac{\pi}{3} + i \sin 10 \frac{\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$Z^{18} = \cos 2\pi + i \sin 2\pi = 1$$

$$\begin{aligned} \left| \frac{Z^{30}}{Z^{18} + 1} \right| &= \left| \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{1 + 1} \right| \\ &= \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2} \\ &= \sqrt{\frac{4}{16}} \\ &= \sqrt{\frac{1}{4}} \\ &= \frac{1}{2} \end{aligned}$$

$$38.) \quad Z^4 - 81 = 0 \rightarrow (Z^2 - 9)(Z^2 + 9) = 0$$

$$Z^2 = 9 \quad \text{หรือ} \quad Z^2 = -9$$

$$Z^2 = \pm 3 \quad \text{หรือ} \quad Z^2 = \pm 3i$$

$$\text{ผลคูณของราก เท่ากับ } (-3)(3)(-3i)(3i) = -81$$

$$39.) \quad \text{ผลบวกคี่ทุกจุด} = 2 \times \text{เส้นเชื่อมทั้งหมด}$$

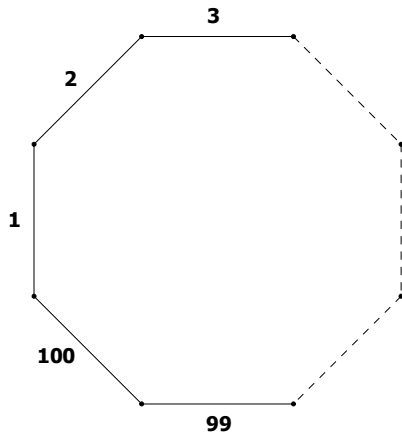
$$\text{ดังนั้น } 3 \times \text{จุดยอด} = 2 \times \text{เส้นเชื่อมทั้งหมด}$$

$$\text{จำนวนจุดยอด} \geq \frac{2 \times 16}{3} = 10 \frac{2}{3}$$

แต่จุดยอด เป็น 1 ไม่ได้เพราะ  $3 \times \text{จุดยอด} = 2 \times \text{เส้นเชื่อมทั้งหมด}$  ซึ่งเป็นเลขคู่

นั่นคือจุดยอดต้องเท่ากับ 12

40.)



ต้นไม้แผ่ทั่วของกราฟวัฏจักร เพียงลบเส้นเชื่อมออก 1 เส้น  
 อยากได้ต้นไม้แผ่ทั่วน้อยที่สุด จึงต้องตัด 100 ออก

$$\text{ดังนั้นผลรวม} = 1 + 2 + 3 + \dots + 99 = \frac{99 \cdot 100}{2} = 4950$$

41.)

$$\begin{aligned} \text{จำนวนที่พ่อและแม่ไม่นั่งติดกัน} &= \text{จำนวนที่นั่งปกติ} - \text{จำนวนวิธีที่พ่อและแม่นั่งติดกัน} \\ &= (6-1)! - 2(5-1)! \\ &= 5! - 2 \cdot 4! \\ &= 4!(5-2) \\ &= 16 \cdot 3 \\ &= 48 \end{aligned}$$

42.)

$$\begin{aligned} \binom{20}{17} &= \binom{19}{17} + \binom{19}{k} \\ \text{จาก } \binom{n}{r} &= \binom{n-1}{r} + \binom{n-1}{r-1} \\ &= 17-1 \\ &= 16 \end{aligned}$$

43.)

$$\begin{aligned} \sum \frac{(x_i - 30)^2}{20} &= 8 \quad \text{จะได้} \quad \sum (x_i - 30)^2 = 160 \\ \sum ((x_i - 29) - 1)^2 &= 160 \\ \sum (x_i - 29)^2 - 2 \sum (x_i - 29) + 1(20) &= 160 \\ \sum (x_i - 29)^2 &= 160 - 20 = 140 \\ (S.D.)^2 &= \frac{140}{20} = 7 \end{aligned}$$

44.)

$$\begin{aligned} \overline{X} \text{ รวม} &= \frac{\overline{N_1 X_1} + \overline{N_2 X_2}}{\overline{N_1} + \overline{N_2}} \\ \text{จะได้} \quad c &= \frac{ax + bd}{a + b} \\ ac + bc &= ax + bd \\ c + \frac{bc}{a} &= x + \frac{bd}{a} \\ x &= c + \frac{b}{a}(c - d) \end{aligned}$$

$$45.) \quad \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{1}{3} \quad \text{-----} > (1)$$

$$\frac{Q_3 - Q_1}{2} = 4 \quad \text{-----} > (2)$$

$$\frac{8}{Q_3 + Q_1} = \frac{1}{3}$$

$$Q_3 + Q_1 = 24 \quad \text{-----} > (3)$$

$$2 \times (2) + 3 \rightarrow 2Q_3 = 32 \rightarrow Q_3 = 16$$

$$\therefore Q_1 = 8$$

$$\frac{Q_3^2 - Q_1^2}{4} = \frac{(16+8)(16-8)}{4} = 48 = 2^4 \cdot 3$$

ดังนั้น จำนวนเต็มบวกที่หาร 48 ลงตัว มี  $(4+1)(1+1)$  เท่ากับ 10 จำนวน

46.)

คะแนน	$f$	$cf$
5-9	3	3
10-14	4	7
15-19	8	15
20-24	9	24
25-29	$a$	$24 + a$
30-34	5	$29 + a$
35-39	3	$32 + a$

$$\therefore P_{80} = 29.5$$

$$\text{ตำแหน่งของ } P_{80} = \frac{80}{100}(32 + a) = \frac{4}{5}(32 + a)$$

$$\text{จะได้ } 24 + a = \frac{4}{5}(32 + a)$$

$$120 + 5a = 128 + 4a$$

$$a = 8$$

47.)

$$\text{จาก } Z = \frac{X - \bar{X}}{S}$$

$$\therefore Z_1 = \frac{X_1 - \bar{X}}{S} \quad \text{-----} > (1)$$

$$Z_2 = \frac{X_2 - \bar{X}}{S} \quad \text{-----} > (2)$$

$$(1) - (2); Z_1 - Z_2 = \frac{X_1 - X_2}{S} = \frac{1}{5} = 0.2$$

48.)

$$\bar{X} = \frac{4+8+10+15+18}{5} = 11 \quad ; \quad Med = 10$$

ดังนั้น ข. ผิด

$$M.D. = \frac{7+3+1+4+7}{5} = \frac{22}{5} = 4.4$$

ดังนั้น ก. ถูก

49.)

ผู้เข้าสอบ	ค่า $Z$			รวม
	วิชาที่ 1	วิชาที่ 2	วิชาที่ 3	
ก	$\frac{30-32}{5} = -\frac{2}{5}$	$\frac{42-40}{10} = \frac{2}{10}$	$\frac{39-40}{5} = -\frac{1}{5}$	$-\frac{3}{10}$
ข	$\frac{32-32}{5} = 0$	$\frac{36-40}{10} = -\frac{4}{10}$	$\frac{43-40}{10} = \frac{3}{10}$	$-\frac{1}{10}$

ดังนั้น ไม่ผ่านทั้ง ก และ ข

50.) สัมประสิทธิ์การแปรผัน  $= \frac{30}{100} = \frac{3}{10}$

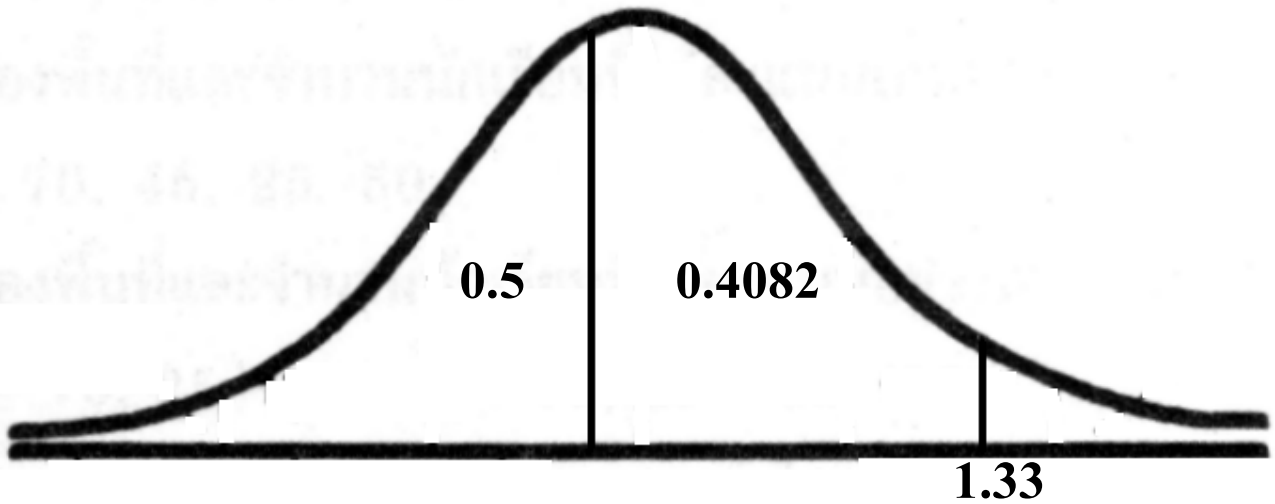
$$\frac{S}{\bar{X}} = \frac{3}{10} \rightarrow \frac{18}{\bar{X}} = \frac{3}{10} \rightarrow \bar{X} = 60$$

$$\therefore Z = \frac{84 - 60}{18}$$

$$= \frac{24}{18}$$

$$= 1.33$$

$$\therefore A = 0.4082$$



ดังนั้นตรงกับเปอร์เซ็นต์ไทล์ที่  $100(0.5 + 0.4082) = 90.82$