

# BASICS OF HEAT TRANSFER

While teaching heat transfer, one of the first questions students commonly ask is the difference between *heat* and *temperature*. Another common question concerns the difference between the subjects of *heat transfer* and *thermodynamics*. Let me begin this chapter by trying to address these two questions.

## 1.1 Difference between heat and temperature

In heat transfer problems, we often interchangeably use the terms *heat* and *temperature*. Actually, there is a distinct difference between the two. *Temperature* is a measure of the amount of energy possessed by the molecules of a substance. It manifests itself as a degree of hotness, and can be used to predict the direction of heat transfer. The usual symbol for temperature is  $T$ . The scales for measuring temperature in SI units are the Celsius and Kelvin temperature scales. *Heat*, on the other hand, is energy in transit. Spontaneously, heat flows from a hotter body to a colder one. The usual symbol for heat is  $Q$ . In the SI system, common units for measuring heat are the Joule and calorie.

## 1.2 Difference between thermodynamics and heat transfer

**Thermodynamics** tells us:

- how much heat is transferred ( $dQ$ )
- how much work is done ( $dW$ )
- final state of the system

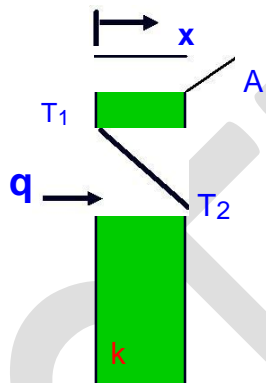
**Heat transfer** tells us:

- how (with what **modes**)  $dQ$  is transferred
- at what **rate**  $dQ$  is transferred
- temperature distribution inside the body



### 1.3 Modes of Heat Transfer

- **Conduction:** An energy transfer across a system boundary due to a temperature difference by the mechanism of inter-molecular interactions. Conduction needs matter and does not require any bulk motion of matter.



Conduction rate equation is described by the Fourier Law:

where:  $q$  = heat flow vector, (W)

$k$  = thermal conductivity, a thermodynamic property of the material.  
(W/m K)

$A$  = Cross sectional area in direction of heat flow.

$\frac{dT}{dx}$  = Gradient of temperature (K/m)

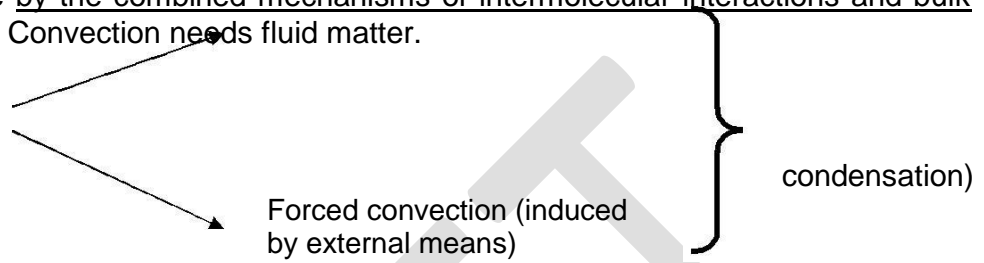
Note: Since this is a vector equation, it is often convenient to work with one component of the vector. For example, in the  $x$  direction:

$$q_x = -k A_x \frac{dT}{dx}$$

In circular coordinates it may be convenient to work in the radial

$$\text{direction: } q_r = -k A_r \frac{dT}{dr}$$

- **Convection:** An energy transfer across a system boundary due to a temperature difference by the combined mechanisms of intermolecular interactions and bulk transport. Convection needs fluid matter.



**Table 1.** Typical values of  $h$  ( $W/m^2K$ )

Free convection	gases: 2 - 25 liquid: 50 - 100
Forced convection	gases: 25 - 250 liquid: 50 - 20,000
Boiling/Condensation	2500 - 100,000

- **Radiation:** Radiation heat transfer involves the transfer of heat by electromagnetic radiation that arises due to the temperature of the body. Radiation does not need matter.

Emissive power of a surface:

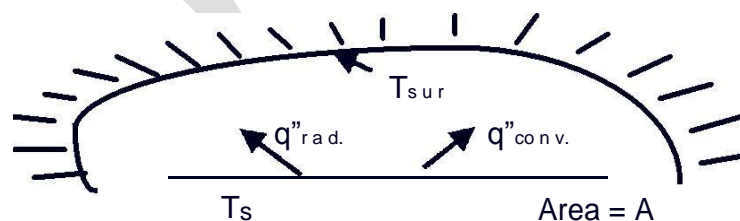
$$E = e\sigma T_s^4 \text{ (W/m}^2\text{)}$$

where:  $e$  = emissivity, which is a surface property ( $e = 1$  is black body)

$\sigma$  = Steffan Boltzman constant =  $5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$ .

$T_s$  = Absolute temperature of the surface (K)

The above equation is derived from Stefan Boltzman law, which describes a gross heat emission rather than heat transfer. The expression for the actual radiation heat transfer rate between surfaces having arbitrary orientations can be quite complex, and will be dealt with in Module 9. However, the rate of radiation heat exchange between a small surface and a large surrounding is given by the following expression:



$$q = \epsilon \cdot \sigma \cdot A \cdot (T_s^4 - T_{sur}^4)$$

where:  $\epsilon$  = Surface Emissivity  
 $A$  = Surface Area

$T_s$  = Absolute temperature of surface. (K)

$T_{sur}$  = Absolute temperature of surroundings.(K)

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## 1.4 Thermal Conductivity, $k$

As noted previously, thermal conductivity is a thermodynamic property of a material. From the State Postulate given in thermodynamics, it may be recalled that thermodynamic properties of pure substances are functions of two independent thermodynamic intensive properties, say temperature and pressure. Thermal conductivity of real gases is largely independent of pressure and may be considered a function of temperature alone. For solids and liquids, properties are largely independent of pressure and depend on temperature alone.

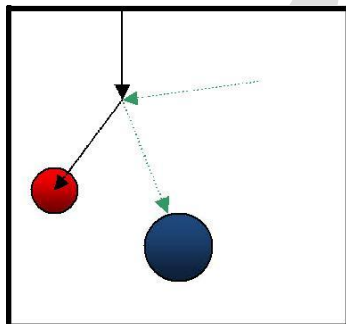
$$k = k(T)$$

Table 2 gives the values of thermal conductivity for a variety of materials.

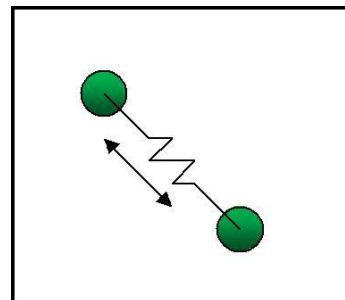
**Table 2.** Thermal Conductivities of Selected Materials at Room Temperature.

Material	Thermal Conductivity, W/m K
Copper	401
Silver	429
Gold	317
Aluminum	237
Steel	60.5
Limestone	2.15
Bakelite	1.4
Water	0.613
Air	0.0263

Let us try to gain an insight into the basic concept of thermal conductivity for various materials. The fundamental concept comes from the molecular or atomic scale activities. Molecules/atoms of various materials gain energy through different mechanisms. Gases, in which molecules are free to move with a mean free path sufficiently large compared to their diameters, possess energy in the form of kinetic energy of the molecules. Energy is gained or lost through collisions/interactions of gas molecules.



*Kinetic energy transfer  
between gas molecules.*



Lattice vibration may be transferred  
between molecules as nuclei  
attract/repel each other.

Solids, on the other hand, have atoms/molecules which are more closely packed which cannot move as freely as in gases. Hence, they cannot effectively transfer energy through these same mechanisms. Instead, solids may exhibit energy through vibration or rotation of the nucleus. Hence, the energy transfer is typically through lattice vibrations.

Another important mechanism in which materials maintain energy is by shifting electrons into higher orbital rings. In the case of electrical conductors the electrons are weakly bonded to the molecule and can drift from one molecule to another, transporting their energy in the process. Hence, flow of electrons, which is commonly observed in metals, is an effective transport mechanism, resulting in a correlation that materials which are excellent electrical conductors are usually excellent thermal conductors.

## ONE DIMENSIONAL STEADY STATE HEAT CONDUCTION

### 2.1 Objectives of conduction analysis:

The primary objective is to determine the temperature field,  $T(x,y,z,t)$ , in a body (i.e. how temperature varies with position within the body)

$T(x,y,z,t)$  depends on:

- Boundary conditions
- Initial condition
- Material properties ( $k$ ,  $c_p$ ,  $\rho$ )
- Geometry of the body (shape, size)

Why we need  $T(x, y, z, t)$ ?

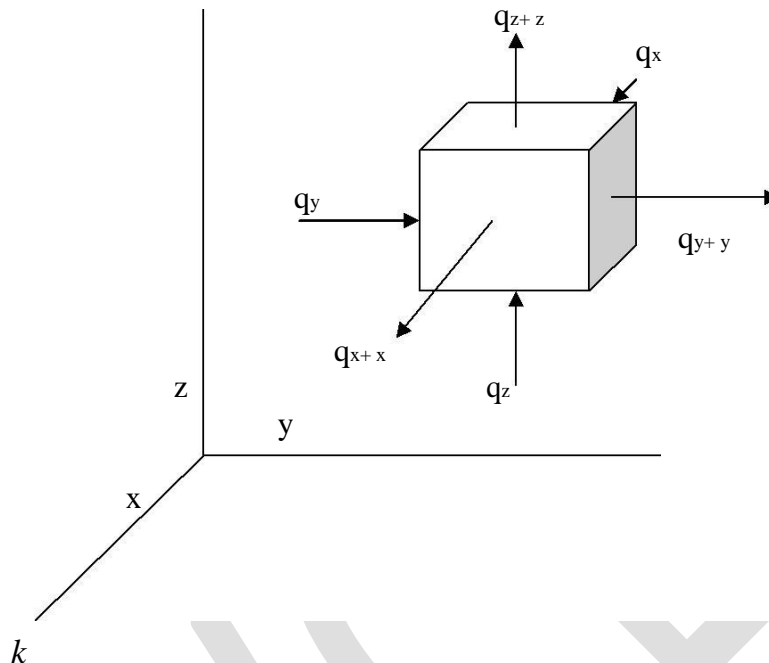
To compute heat flux at any location (using Fourier's eqn.)

Compute thermal stresses, expansion, deflection due to temp. Etc.

Design insulation thickness

Chip temperature calculation

Heat treatment of metals

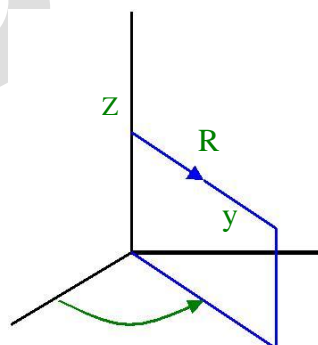


The vector form of this equation is quite compact and is the most general form. However, we often find it convenient to expand the spatial derivative in specific coordinate systems:

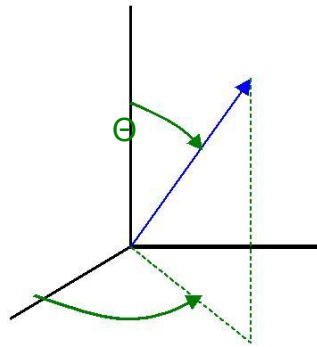
Cartesian Coordinates

$$\frac{1}{a} \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{q}{k}$$

Circular Coordinates







In each equation the dependent variable,  $T$ , is a function of 4 independent variables,  $(x, y, z, \tau)$ ;  $(r, \theta, \tau)$ ;  $(r, \phi, \theta, \tau)$  and is a 2<sup>nd</sup> order, partial differential equation. The solution of such equations will normally require a numerical solution. For the present, we shall simply look at the simplifications that can be made to the equations to describe specific problems.

**Steady State:** Steady state solutions imply that the system conditions are not changing with time. Thus  $\partial T / \partial \tau = 0$ .

**One dimensional:** If heat is flowing in only one coordinate direction, then it follows that there is no temperature gradient in the other two directions. Thus the two partials associated with these directions are equal to zero.

**Two dimensional:** If heat is flowing in only two coordinate directions, then it follows that there is no temperature gradient in the third direction. Thus, the partial derivative associated with this third direction is equal to zero.

**No Sources:** If there are no volumetric heat sources within the system then the term,

$$q = 0.$$

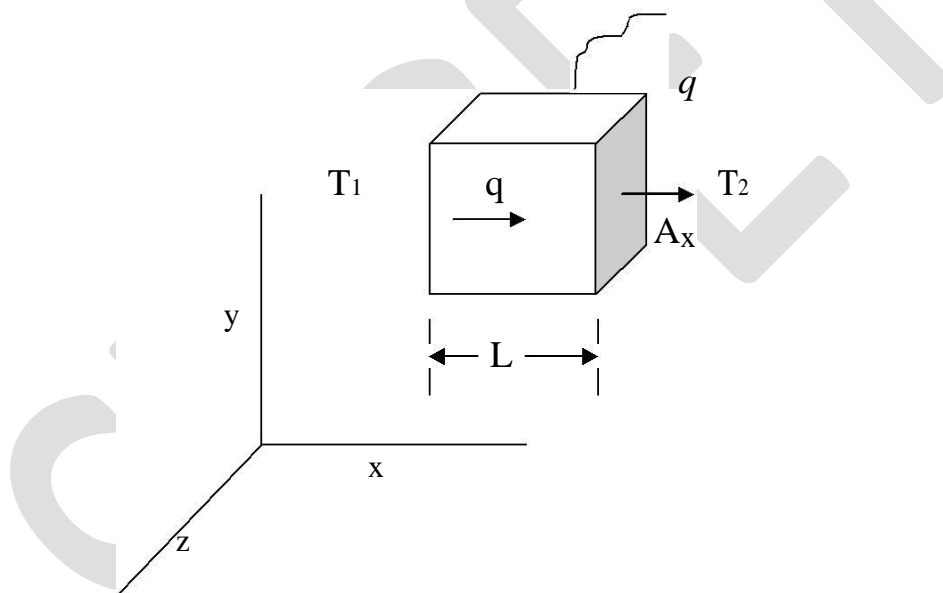
Note that the equation is 2<sup>nd</sup> order in each coordinate direction so that integration will result in 2 constants of integration. To evaluate these constants two boundary conditions will be required for each coordinate direction.

## • Boundary and Initial Conditions

The objective of deriving the heat diffusion equation is to determine the temperature distribution within the conducting body.

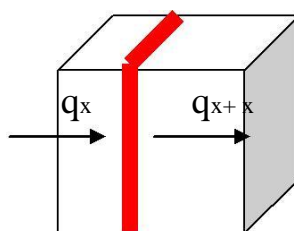
- We have set up a differential equation, with  $T$  as the dependent variable. The solution will give us  $T(x,y,z)$ . Solution depends on boundary conditions (BC) and initial conditions (IC).
- How many BC's and IC's ?
  - Heat equation is second order in spatial coordinate. Hence, 2 BC's needed for each coordinate.
    - \* 1D problem: 2 BC in x-direction
    - \* 2D problem: 2 BC in x-direction, 2 in y-direction
    - \* 3D problem: 2 in x-dir., 2 in y-dir., and 2 in z-dir.
  - Heat equation is first order in time. Hence one IC needed.

## 2.4 Heat Diffusion Equation for a One Dimensional System



Consider the system shown above. The top, bottom, front and back of the cube are insulated, so that heat can be conducted through the cube only in the  $x$  direction. The internal heat generation per unit volume is  $q$  ( $\text{W/m}^3$ ).

Consider the heat flow through a differential element of the cube.



## 2.4 One Dimensional Steady State Heat Conduction

### The plane wall:

The differential equation governing heat diffusion is:  $\frac{d}{dx} \left( k \frac{dT}{dx} \right) = 0$

With constant k, the above equation may be integrated twice to obtain the general solution:

$$T(x) = C_1 x + C_2$$

where  $C_1$  and  $C_2$  are constants of integration. To obtain the constants of integration, we apply the boundary conditions at  $x = 0$  and  $x = L$ , in which case

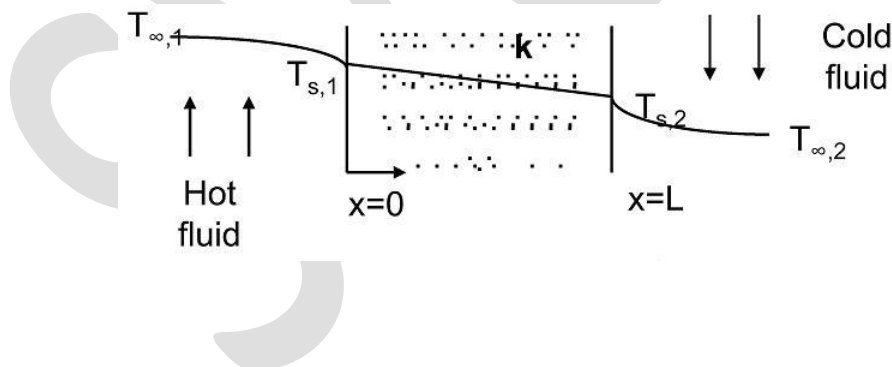
$$T(0) = T_{s,1} \quad \text{and} \quad T(L) = T_{s,2}$$

Once the constants of integration are substituted into the general equation, the temperature distribution is obtained:

$$T(x) = (T_{s,2} - T_{s,1}) \frac{x}{L} + T_{s,1}$$

The heat flow rate across the wall is given by:

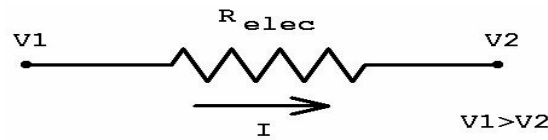
$$q_x = -kA \frac{dT}{dx} = \frac{kA}{L} (T_{s,1} - T_{s,2}) = \frac{T_{s,1} - T_{s,2}}{L/kA}$$



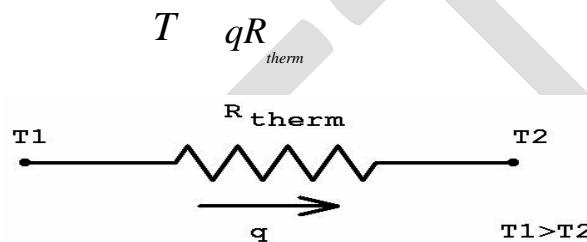
### Thermal resistance (electrical analogy):

Physical systems are said to be analogous if that obey the same mathematical equation. The above relations can be put into the form of Ohm's law:

$$V = IR_{\text{elec}}$$



Using this terminology it is common to speak of a thermal resistance:



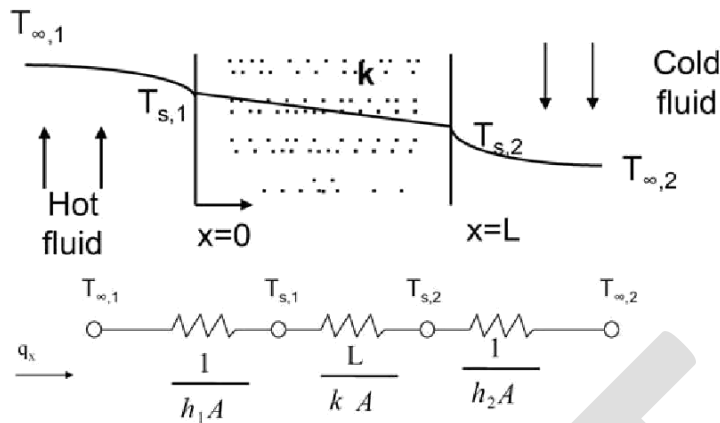
A thermal resistance may also be associated with heat transfer by convection at a surface. From Newton's law of cooling,

$$q = hA(T_s - T_\infty)$$

the thermal resistance for convection is then

$$R_{t, \text{conv}} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$

Applying thermal resistance concept to the plane wall, the equivalent thermal circuit for the plane wall with convection boundary conditions is shown in the figure below



The heat transfer rate may be determined from separate consideration of each element in the network. Since  $q_x$  is constant throughout the network, it follows that

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A}$$

In terms of the overall temperature difference  $T_{\infty,1} - T_{\infty,2}$ , and the total thermal resistance  $R_{tot}$ , the heat transfer rate may also be expressed as

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}}$$

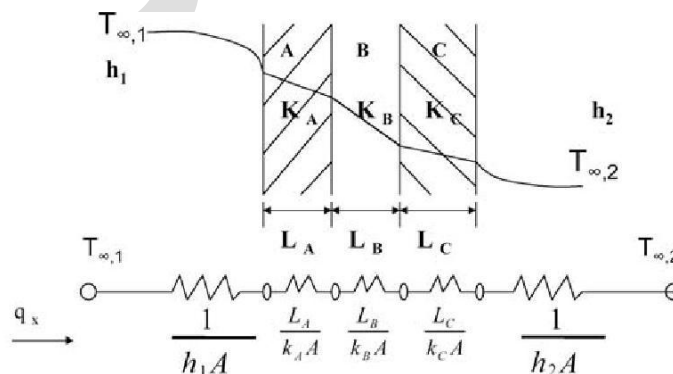
Since the resistance are in series, it follows that

$$R_{tot} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

### Composite walls:

#### Thermal Resistances in Series:

Consider three blocks, A, B and C, as shown. They are insulated on top, bottom, front and back. Since the energy will flow first through block A and then through blocks B and C, we say that these blocks are thermally in a series arrangement.



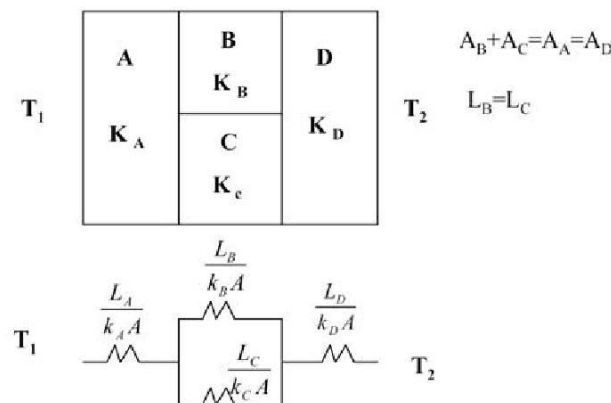
The steady state heat flow rate through the walls is given by:

$$q_x = \frac{T_1 - T_2}{R_{tot}} = \frac{T_1 - T_2}{\frac{1}{h_1 A} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{L_C}{k_C A} + \frac{1}{h_2 A}} UA(T_1 - T_2)$$

where  $U = \frac{1}{R_{tot} A}$  is the overall heat transfer coefficient. In the above case, U is expressed as

$$U = \frac{1}{\frac{1}{h_1} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_2}}$$

Series-parallel arrangement:

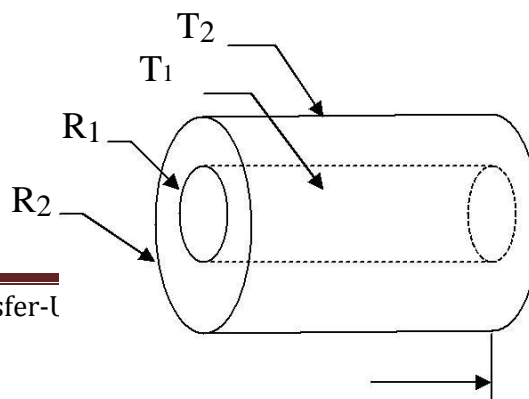


The following assumptions are made with regard to the above thermal resistance model:

- 1) Face between B and C is insulated.
- 2) Uniform temperature at any face normal to X.

### 1-D radial conduction through a cylinder:

One frequently encountered problem is that of heat flow through the walls of a pipe or through the insulation placed around a pipe. Consider the cylinder shown. The pipe is either insulated on the ends or is of sufficient length, L, that heat losses through the ends is negligible. Assume no heat sources within the wall of the tube. If  $T_1 > T_2$ , heat will flow outward, radially, from the inside radius,  $R_1$ , to the outside radius,  $R_2$ . The process will be described by the Fourier Law.





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The differential equation governing heat diffusion is:

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) = 0$$

With constant k, the solution is

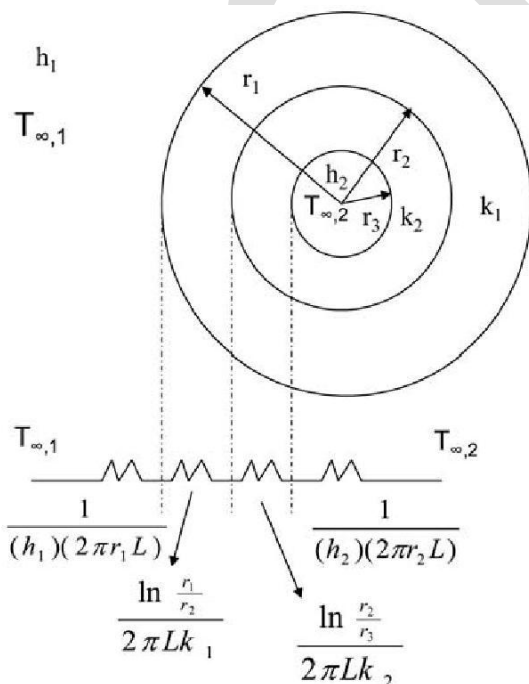
The heat flow rate across the wall is given by:

$$q_x = kA \frac{dT}{dx} = \frac{kA}{L} (T_{s,1} - T_{s,2}) = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,1} - T_{s,2}}{\ln \frac{r_1}{r_2}} \cdot \frac{2\pi L k}{\ln \frac{r_1}{r_2}}$$

Hence, the thermal resistance in this case can be expressed as:

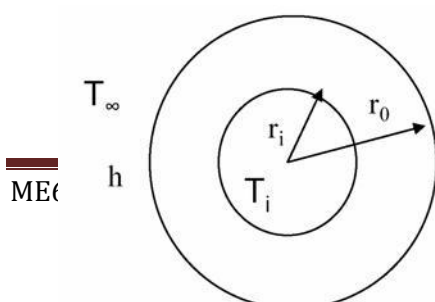
$$R = \frac{\ln \frac{r_1}{r_2}}{2\pi L k}$$

### Composite cylindrical walls:



$$q_r = \frac{T_{\infty,2} - T_{\infty,1}}{\sum R_i}$$

### Critical Insulation Thickness :



$$R = \frac{\ln \left( \frac{r_0}{r_i} \right)}{2\pi L k} + \frac{1}{h_0}$$



$$2 kL \quad (2 r_o L)h$$

Insulation thickness :  $r_o - r_i$

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Objective : decrease  $q$  , increase  $R_{tot}$

Vary  $r_o$  ; as  $r_o$  increases, first term increases, second term decreases.

This is a maximum – minimum problem. The point of extrema can be found by setting

$$\frac{dR_{tot}}{dr_o} = 0$$

$$\text{or, } \frac{1}{2kr_oL} - \frac{1}{2hr_o^2} = 0$$

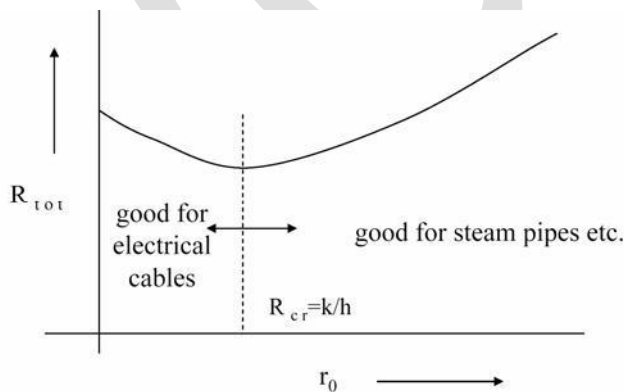
$$\text{or, } \frac{k}{r_o} = \frac{h}{r_o^2}$$

In order to determine if it is a maxima or a minima, we make the second derivative zero:

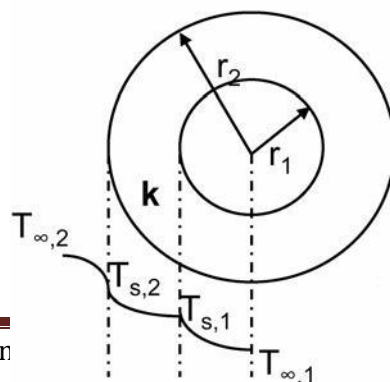
$$\frac{d^2 R_{tot}}{dr_o^2} < 0 \text{ at } r_o = \frac{k}{h}$$

$$\frac{d^2 R_{tot}}{dr_o^2} = \frac{1}{2kr_o^2L} - \frac{1}{r_o^3hL} \Bigg|_{r_o = \frac{k}{h}} = \frac{h^2}{2Lk^3} < 0$$

Minimum  $q$  at  $r_o = (k/h) = r_{cr}$  (critical radius)



### 1-D radial conduction in a sphere:



$$\frac{1}{r^2} \frac{d}{dr} \left( kr^2 \frac{dT}{dr} \right) = 0$$

$$q_r = -kA \frac{dT}{dr} = -k(4\pi r^2) \frac{dT}{dr}$$

$$q_r = \frac{4\pi k r^2}{r_1 - r_2} (T_{s,1} - T_{s,2})$$

$$R_{t,cond} = \frac{1/r_1 - 1/r_2}{4k}$$

## 2.6 Summary of Electrical Analogy

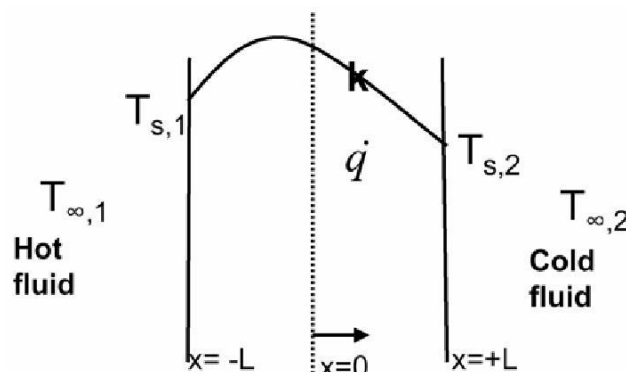
System	Current	Resistance	Potential Difference
Electrical	I	R	V
Cartesian Conduction	q	$\frac{L}{kA}$	T
Cylindrical Conduction	q	$\frac{\ln(r_2/r_1)}{2\pi kL}$	T
Conduction through sphere	q	$\frac{1/r_1 - 1/r_2}{4k}$	T
Convection	q	$\frac{1}{hA_s}$	T

## 2.7 One-Dimensional Steady State Conduction with Internal Heat Generation

Applications: current carrying conductor, chemically reacting systems, nuclear reactors.

Energy generated per unit volume is given by  $q'''$

Plane wall with heat source: Assumptions: 1D, steady state, constant k, uniform  $q'''$



$$\frac{d^2 T}{dx^2} = \frac{q_0}{k}$$

$k$

$$\text{Boundary cond. : } T(x=L) = T_{s,1} \quad T(x=L) = T_{s,2}$$

$$\text{Solution: } T = \frac{q_0}{2k} x^2 + C_1 x + C_2$$

Use boundary conditions to find  $C_1$  and  $C_2$

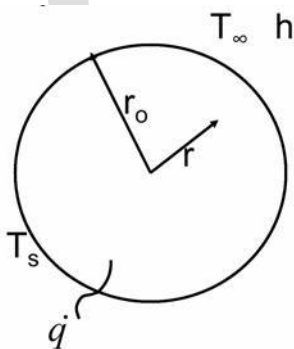
$$\text{Final solution : } T = \frac{q_0}{2k} x^2 + \frac{T_{s,2} - T_{s,1}}{L} x + \frac{T_{s,2} + T_{s,1}}{2}$$

$$\text{Heat flux : } q_x = k \frac{dT}{dx}$$

**Note:** From the above expressions, it may be observed that the solution for temperature is no longer linear. As an exercise, show that the expression for heat flux is no longer independent of  $x$ . Hence thermal resistance concept is not correct to use when there is internal heat generation.

Cylinder with heat source: Assumptions: 1D, steady state, constant  $k$ , uniform  $q$

Start with 1D heat equation in cylindrical co-ordinates



$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) = \frac{q}{k}$$

$$\text{Boundary cond. : } T(r_0) = T_s \quad \frac{dT}{dr} \bigg|_{r=0} = 0$$

$$\text{Solution : } T(r) = \frac{q}{4k} (r_0^2 - r^2) + T_s$$

**Exercise:**  $T_s$  may not be known. Instead,  $T_\infty$  and  $h$  may be specified. Eliminate  $T_s$ , using  $T_\infty$  and  $h$ .

## Extended Surface Heat Transfer

### 3.1 Introduction:

Convection: Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law:  $q = hA(T_s - T)$ , where  $T_s$  is the surface temperature and  $T$  is the fluid temperature. Therefore, to increase the convective heat transfer, one can

- Increase the temperature difference ( $T_s - T$ ) between the surface and the fluid.
- Increase the convection coefficient  $h$ . This can be accomplished by increasing the fluid flow over the surface since  $h$  is a function of the flow velocity and the higher the velocity, the higher the  $h$ . Example: a cooling fan.
- Increase the contact surface area  $A$ . Example: a heat sink with fins.

Many times, when the first option is not in our control and the second option (i.e. increasing  $h$ ) is already stretched to its limit, we are left with the only alternative of increasing the effective surface area by using fins or extended surfaces. Fins are protrusions from the base surface into the cooling fluid, so that the extra surface of the protrusions is also in contact with the fluid. Most of you have encountered cooling fins on air-cooled engines (motorcycles, portable generators, etc.), electronic equipment (CPUs), automobile radiators, air conditioning equipment (condensers) and elsewhere.

### 3.2 Extended surface analysis:

In this module, consideration will be limited to steady state analysis of rectangular or pin fins of constant cross sectional area. Annular fins or fins involving a tapered cross section may be analyzed by similar methods, but will involve solution of more complicated equations which result. Numerical methods of integration or computer programs can be used to advantage in such cases.

We start with the General Conduction Equation:

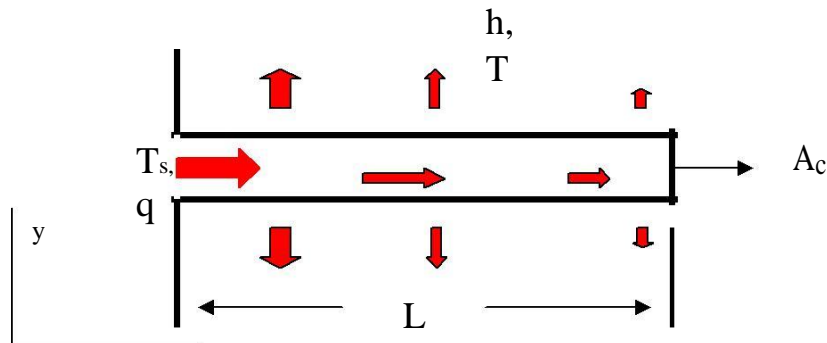
$$\frac{1}{d} \frac{dT}{dx} \bigg|_{\text{system}} = \frac{2T}{k} \quad (1)$$

After making the assumptions of Steady State, One-Dimensional Conduction, this equation reduces to the form:

$$\frac{d^2 T}{dx^2} + \frac{q}{k} = 0 \quad (2)$$

This is a second order, ordinary differential equation and will require 2 boundary conditions to evaluate the two constants of integration that will arise.

Consider the cooling fin shown below:

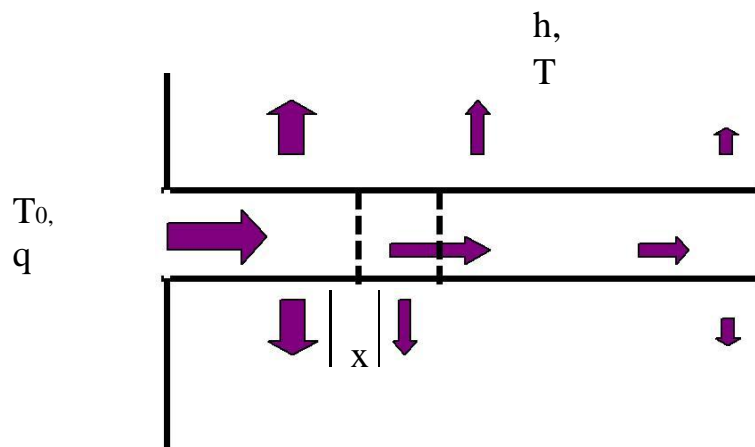


The fin is situated on the surface of a hot surface at  $T_s$  and surrounded by a coolant at temperature  $T$ , which cools with convective coefficient,  $h$ . The fin has a cross sectional area,  $A_c$ , (This is the area through which heat is conducted.) and an overall length,  $L$ .

Note that as energy is conducted down the length of the fin, some portion is lost, by convection, from the sides. Thus the heat flow varies along the length of the fin.

We further note that the arrows indicating the direction of heat flow point in both the  $x$  and  $y$  directions. This is an indication that this is truly a two- or three-dimensional heat flow, depending on the geometry of the fin. However, quite often, it is convenient to analyse a fin by examining an equivalent one-dimensional system. The equivalent system will involve the introduction of heat sinks (negative heat sources), which remove an amount of energy equivalent to what would be lost through the sides by convection.

Consider a differential length of the fin.



Across this segment the heat loss will be  $h (P \cdot x) (T - T_c)$ , where  $P$  is the perimeter around the fin. The equivalent heat sink would be  $q = A \cdot x \cdot h_c$ .



Equating the heat source to the convective loss:

$$q = \frac{h P (T_c - T_\infty)}{A_c} \quad (3)$$

Substitute this value into the General Conduction Equation as simplified for One-Dimension, Steady State Conduction with Sources:

$$\frac{d^2 T}{dx^2} + \frac{h P}{k A_c} (T_c - T_\infty) = 0 \quad (4)$$

which is the equation for a fin with a constant cross sectional area. This is the Second Order Differential Equation that we will solve for each fin analysis. Prior to solving, a couple of simplifications should be noted. First, we see that  $h$ ,  $P$ ,  $k$  and  $A_c$  are all independent of  $x$  in the defined system (They may not be constant if a more general analysis is desired.). We replace this ratio with a constant. Let

$$m^2 = \frac{h P}{k A_c} \quad (5)$$

then:

$$\frac{d^2 T}{dx^2} + m^2 (T_c - T_\infty) = 0 \quad (6)$$

Next we notice that the equation is non-homogeneous (due to the  $T$  term). Recall that non-homogeneous differential equations require both a general and a particular solution. We can make this equation homogeneous by introducing the temperature relative to the surroundings:

$$T - T_\infty \quad (7)$$

Differentiating this equation we find:

$$\frac{dT}{dx} = \frac{dT}{dx} \quad (8)$$

Differentiate a second time:

$$\frac{d^2 T}{dx^2} = \frac{d^2 T}{dx^2} \quad (9)$$

Substitute into the Fin Equation:

$$\frac{d^2}{dx^2} (T - T_\infty) + m^2 (T_c - T_\infty) = 0 \quad (10)$$

This equation is a Second Order, Homogeneous Differential Equation.

So that the energy flowing through the base of the fin is:

$$q = \sqrt{hPkA_c} \theta_0 \tanh(mL) \quad (36)$$

If we compare this result with that for the very long fin, we see that the primary difference in form is in the hyperbolic tangent term. That term, which always results in a number equal to or less than one, represents the reduced heat loss due to the shortening of the fin.

*Other tip conditions:*

We have already seen two tip conditions, one being the long fin and the other being the insulated tip. Two other possibilities are usually considered for fin analysis: (i) a tip subjected to convective heat transfer, and (ii) a tip with a prescribed temperature. The expressions for temperature distribution and fin heat transfer for all the four cases are summarized in the table below.

**Table 3.1**

Case	Tip Condition	Temp. Distribution	Fin heat transfer
A	Convection heat transfer: $h(L) = -k(d/dx)_{x=L}$	$\frac{\cosh m(L-x) \left( \frac{h}{mk} \right) \sinh m(L-x)}{\cosh mL \left( \frac{h}{mk} \right) \sinh mL}$	$M_0 \frac{\sinh mL \left( \frac{h}{mk} \right) \cosh mL}{\cosh mL \left( \frac{h}{mk} \right) \sinh mL}$
B	Adiabatic $(d/dx)_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M_0 \tanh mL$
C	Given temperature: $(L) = T_L$	$\frac{\left( \frac{T_L - T_\infty}{T_0 - T_\infty} \right) \sinh m(L-x)}{\sinh mL}$	$M_0 \frac{(\cosh mL - \frac{L}{b})}{\sinh mL}$
D	Infinitely long fin $(L) = 0$	$e^{-mx}$	$M_0$

### 3.4 Fin Effectiveness

How effective a fin can enhance heat transfer is characterized by the fin effectiveness,  $\eta_f$ , which is as the ratio of fin heat transfer and the heat transfer without the fin. For an adiabatic fin:

$$\eta_f = \frac{q_f}{q_c} = \frac{q_f}{hA_c(T_b - T_\infty)} = \frac{\sqrt{hPkA_c} \tanh(mL)}{hA_c} = \sqrt{\frac{kP}{hA_c}} \tanh(mL) \quad (37)$$

If the fin is long enough,  $mL > 2$ ,  $\tanh(mL) \rightarrow 1$ , and hence it can be considered as infinite fin (case D in Table 3.1). Hence, for long fins,

SVCE

$$f = \sqrt{\frac{kP}{hA_c}} = \sqrt{\frac{k}{h} \frac{P}{A_c}} \quad (38)$$

In order to enhance heat transfer,  $f$  should be greater than 1 (In case  $f < 1$ , the fin would have no purpose as it would serve as an insulator instead). However  $f^2$  is considered unjustifiable because of diminishing returns as fin length increases.

To increase  $f$ , the fin's material should have higher thermal conductivity,  $k$ . It seems to be counterintuitive that the lower convection coefficient,  $h$ , the higher  $f$ . Well, if  $h$  is very high, it is not necessary to enhance heat transfer by adding heat fins. Therefore, heat fins are more effective if  $h$  is low.

#### Observations:

If fins are to be used on surfaces separating gas and liquid, fins are usually placed on the gas side. (Why?)

$P/A_c$  should be as high as possible. Use a square fin with a dimension of  $W$  by  $W$  as

an example:  $P=4W$ ,  $A_c=W^2$ ,  $P/A_c=(4/W)$ . The smaller the  $W$ , the higher is the  $P/A_c$ , and the higher the  $f$ . Conclusion: It is preferred to use thin and closely spaced (to increase the total number) fins.

The effectiveness of a fin can also be characterized by

$$\epsilon_f = \frac{q_f}{q_{cb}} = \frac{(T_b - T_\infty) / R_{t,f}}{(T_b - T_\infty) / R_{t,h}} = \frac{R_{t,h}}{R_{t,f}} \quad (39)$$

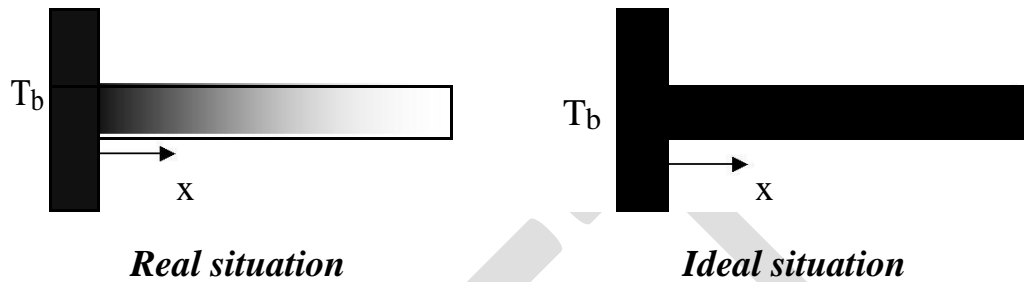
It is a ratio of the thermal resistance due to convection to the thermal resistance of a fin. In order to enhance heat transfer, the fin's resistance should be lower than the resistance due only to convection.

### 3.5 Fin Efficiency

The fin efficiency is defined as the ratio of the energy transferred through a real fin to that transferred through an ideal fin. An ideal fin is thought to be one made of a perfect or infinite conductor material. A perfect conductor has an infinite thermal conductivity so that the entire fin is at the base material temperature.

$$\frac{q_{real}}{q_{ideal}} = \frac{h P k A_c L \tanh(m L)}{h P L} \quad (40)$$

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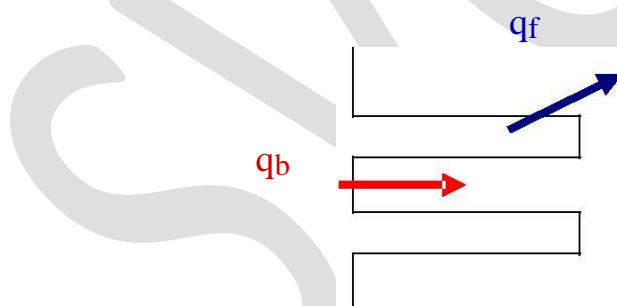


Simplifying equation (40):

$$\frac{k A_c L}{h P} \frac{\tanh(m L)}{L} = \frac{\tanh(m L)}{m L}$$

### Overall Fin Efficiency:

Overall fin efficiency for an array of fins



**Define terms:** Ab: base area exposed to

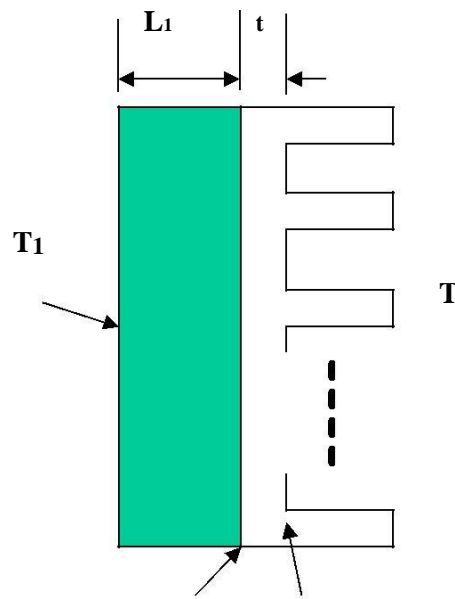
coolant Af: surface area of a single fin

At: total area including base area and total finned surface,  $A_t = A_b + N A_f$

N: total number of fins

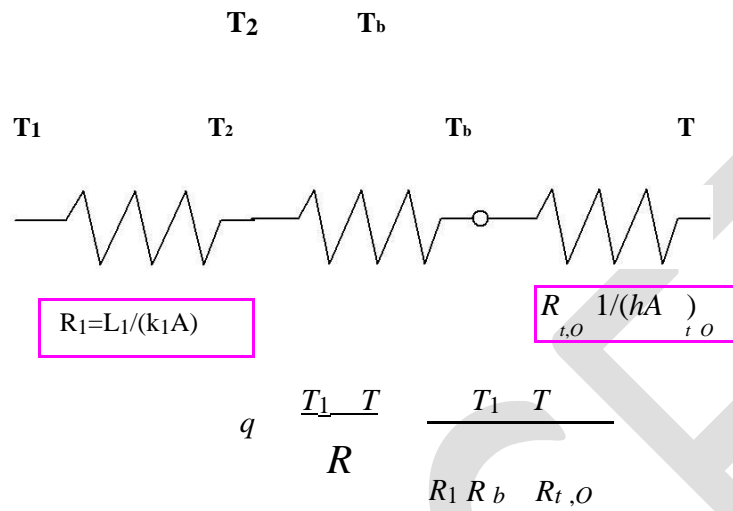
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### Thermal Resistance Concept:



$$A = A_b + N A_{b,f}$$

$$R_b = t / (k b A)$$



# CONVECTION

## Objectives of convection analysis:

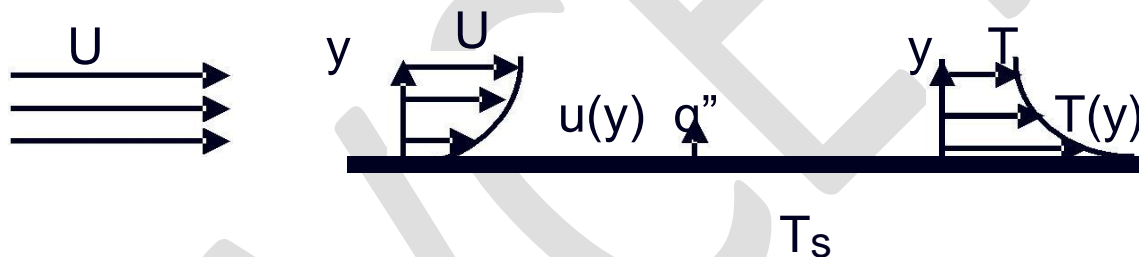
**Main purpose of convective heat transfer analysis is to determine:**

- flow field
- temperature field in fluid
- heat transfer coefficient,  $h$

## How do we determine ?

Consider the process of convective cooling, as we pass a cool fluid past a heated wall. This process is described by Newton's law of Cooling:

$$q = h \cdot A \cdot (T_s - T)$$



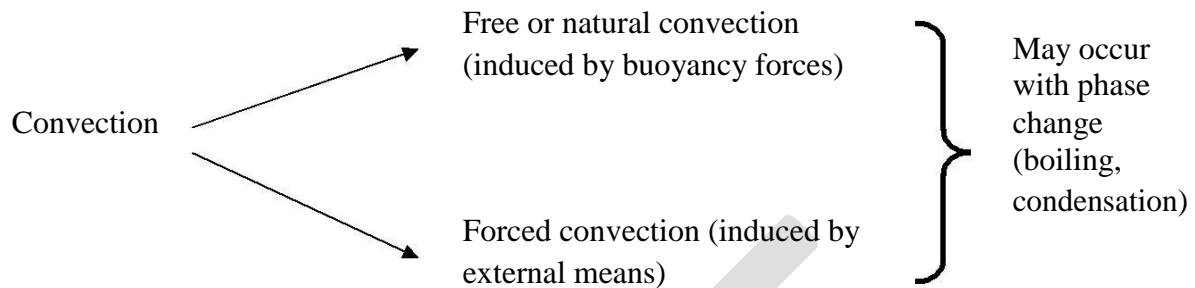
Near any wall a fluid is subject to the no slip condition; that is, there is a stagnant sub layer. Since there is no fluid motion in this layer, heat transfer is by conduction in this region. Above the sub layer is a region where viscous forces retard fluid motion; in this region some convection may occur, but conduction may well predominate. A careful analysis of this region allows us to use our conductive analysis in analyzing heat transfer. This is the basis of our convective theory.

At the wall, the convective heat transfer rate can be expressed as the heat flux.

*The expression shows that in order to determine  $h$ , we must first determine the temperature distribution in the thin fluid layer that coats the wall.*



## Classes of Convective Flows



- extremely diverse
- several parameters involved (fluid properties, geometry, nature of flow, phases etc)
- systematic approach required
- classify flows into certain types, based on certain parameters
- identify parameters governing the flow, and group them into **meaningful non-dimensional numbers**
- need to understand the physics behind each phenomenon

### Common classifications:

#### A. *Based on geometry:*

External flow / Internal flow

#### B. *Based on driving mechanism*

Natural convection / forced convection / mixed convection

#### C. *Based on number of phases*

Single phase / multiple

#### phase D. *Based on nature of flow*

Laminar / turbulent

. Typical values of  $h$  ( $\text{W/m}^2\text{K}$ )

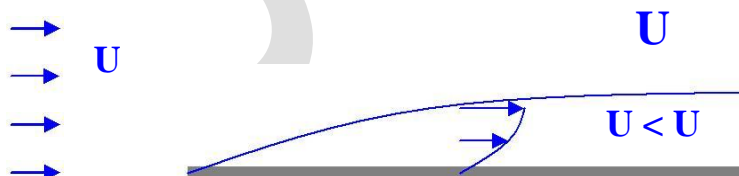
Free convection	gases: 2 - 25 liquid: 50 - 100
Forced convection	gases: 25 - 250 liquid: 50 - 20,000
Boiling/Condensation	2500 - 100,000

### How to solve a convection problem ?

- Solve governing equations along with boundary conditions
- Governing equations include
  1. conservation of mass
  2. conservation of momentum
  3. conservation of energy
- In Conduction problems, only (3) is needed to be solved. Hence, only *few parameters* are involved
- In Convection, all the governing equations need to be solved. large number of parameters can be involved

### FORCED CONVECTION: external flow (over flat plate)

An internal flow is surrounded by solid boundaries that can restrict the development of its boundary layer, for example, a pipe flow. An external flow, on the other hand, are flows over bodies immersed in an unbounded fluid so that the flow boundary layer can grow freely in one direction. Examples include the flows over airfoils, ship hulls, turbine blades, etc



- Fluid particle adjacent to the solid surface is at rest
- These particles act to retard the motion of adjoining layers
- boundary layer effect

Inside the boundary layer, we can apply the following conservation principles:

**Momentum balance:** inertia forces, pressure gradient, viscous forces, body forces

**Energy balance:** convective flux, diffusive flux, heat generation, energy storage

## Forced Convection Correlations

Since the heat transfer coefficient is a direct function of the temperature gradient next to the wall, the physical variables on which it depends can be expressed as follows:

$h=f(\text{fluid properties, velocity field, geometry, temperature etc.})$

As the function is dependent on several parameters, the heat transfer coefficient is usually expressed in terms of **correlations involving pertinent non-dimensional numbers**.

Forced convection: **Non-dimensional groupings**

- **Nusselt No.**  $Nu = hx / k = (\text{convection heat transfer strength}) / (\text{conduction heat transfer strength})$
- **Prandtl No.**  $Pr = \nu / \alpha = (\text{momentum diffusivity}) / (\text{thermal diffusivity})$
- **Reynolds No.**  $Re = Ux / \nu = (\text{inertia force}) / (\text{viscous force})$

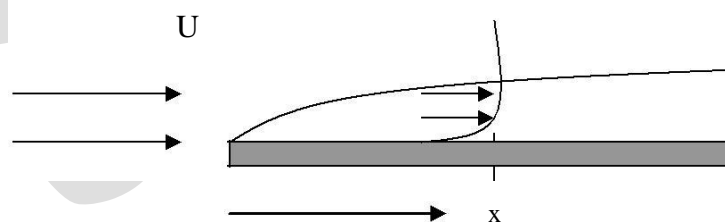
Viscous force provides the dampening effect for disturbances in the fluid. If dampening is strong enough **laminar flow**

Otherwise, instability **turbulent flow** **critical Reynolds number**

For forced convection, the heat transfer correlation can be expressed as

$$Nu=f(Re, Pr)$$

The convective correlation for laminar flow across a flat plate heated to a constant wall temperature is:



$$Nu_x = 0.323 \cdot Re_x^{1/2} \cdot Pr^{1/3}$$

where

$$Nu_x = hx/k$$

$$Re_x = (Ux)/\nu$$

$$Pr = c_p \mu / k$$

### Physical Interpretation of Convective Correlation

The Reynolds number is a familiar term to all of us, but we may benefit by considering what the ratio tells us. Recall that the thickness of the dynamic boundary layer,  $\delta$ , is proportional to the distance along the plate,  $x$ .

$$Re_x = (U \cdot x) / \nu = (\rho U^2 x) / \mu$$

The numerator is a mass flow per unit area times a velocity; i.e. a momentum flow per unit area. The denominator is a viscous stress, i.e. a viscous force per unit area. The ratio represents the ratio of momentum to viscous forces. If viscous forces dominate, the flow will be laminar; if momentum dominates, the flow will be turbulent.

## Physical Meaning of Prandtl Number

The Prandtl number was introduced earlier.

If we multiply and divide the equation by the fluid density,  $\rho$ , we obtain:

$$\text{Pr} = (\mu / \rho) / (k / \rho c_p) = \mu c_p / k$$

The Prandtl number may be seen to be a ratio reflecting the ratio of the rate that viscous forces penetrate the material to the rate that thermal energy penetrates the material. As a consequence the Prandtl number is proportional to the rate of growth of the two boundary layers:

$$\delta_t = \delta \cdot \text{Pr}^{1/3}$$

## Physical Meaning of Nusselt Number

The Nusselt number may be physically described as

$$\text{well. } \text{Nu}_x = h x / k$$

If we recall that the thickness of the boundary layer at any point along the surface,  $x$ , is also a function of  $x$  then

$$\text{Nu}_x = h x / k = (h / k A) / (1 / h A)$$

We see that the Nusselt may be viewed as the ratio of the conduction resistance of a material to the convection resistance of the same material.

Students, recalling the Biot number, may wish to compare the two so that they may distinguish the two.

$$\text{Nu}_x = h x / k_{\text{fluid}}$$

$$\text{Bi}_x = h x / k_{\text{solid}}$$

The denominator of the Nusselt number involves the thermal conductivity of the **fluid** at the solid-fluid convective interface; The denominator of the Biot number involves the thermal conductivity of the **solid** at the solid-fluid convective interface.

## Local Nature of Convective Correlation

Consider again the correlation that we have developed for laminar flow over a flat plate at constant wall temperature

$$\text{Nu}_x = 0.323 \cdot \text{Re}_x^{1/2} \cdot \text{Pr}^{1/3}$$

To put this back into dimensional form, we replace the Nusselt number by its equivalent,  $hx/k$  and take the  $x/k$  to the other side:

$$h = 0.323 \cdot (k/x) \text{Re}_x^{1/2} \cdot \text{Pr}^{1/3}$$

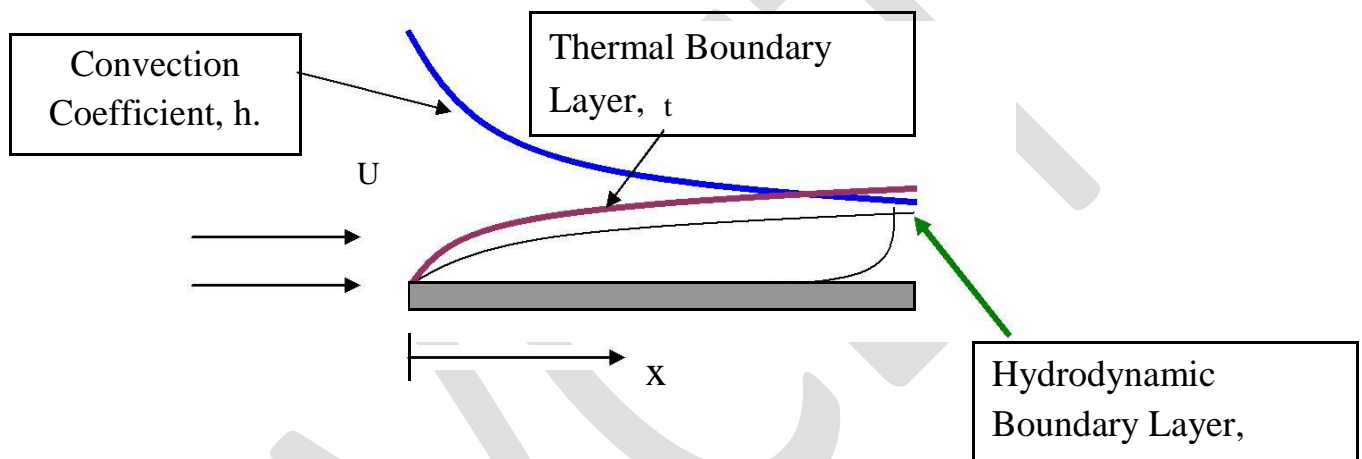
$\text{Pr}^{1/3}$  Now expand the Reynolds number

$$h = 0.323 \cdot (k/x) [(U x) / \nu]^{1/2} \cdot \text{Pr}^{1/3}$$

We proceed to combine the  $x$  terms:

$$h = 0.323 \cdot k [(U) / (\nu x)]^{1/2} \cdot \text{Pr}^{1/3}$$

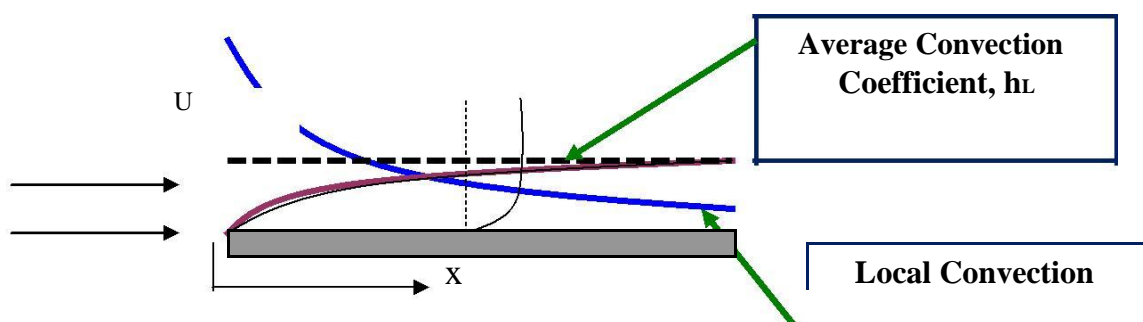
And see that the convective coefficient decreases with  $x^{1/2}$ .



We see that as the boundary layer thickens, the convection coefficient decreases. Some designers will introduce a series of “trip wires”, i.e. devices to disrupt the boundary layer, so that the buildup of the insulating layer must begin anew. This will result in regular “thinning” of the boundary layer so that the convection coefficient will remain high.

### Averaged Correlations

If one were interested in the total heat loss from a surface, rather than the temperature at a point, then they may well want to know something about average convective coefficients.



**Coefficient,  $h_x$ .**

The desire is to find a correlation that provides an overall heat transfer rate:

$$Q = h_L A [T_{\text{wall}} - T] = \int_{x_{\text{wall}}}^L h [T_{\text{wall}} - T] dA$$

where  $h_x$  and  $h_L$ , refer to local and average convective coefficients, respectively.

Compare the second and fourth equations where the area is assumed to be equal to  $A = (L)$ :

$$h_L L [T_{\text{wall}} - T] = \int_0^L h_x [T_{\text{wall}} - T] dx$$

Since the temperature difference is constant, it may be taken outside of the integral and cancelled:

$$h_L L = \int_0^L h_x dx$$

This is a general definition of an integrated average.

$$Nu_L = 0.646 Re_L^{0.5} Pr^{1/3}$$

This is our average correlation for laminar flow over a flat plate with constant wall temperature.

### Reynolds Analogy

In the development of the boundary layer theory, one may notice the strong relationship between the dynamic boundary layer and the thermal boundary layer. Reynold's noted the strong correlation and found that fluid friction and convection coefficient could be related. This is known as the Reynolds Analogy.

**Conclusion from Reynold's analogy:** Knowing the frictional drag, we know the Nusselt Number. If the drag coefficient is increased, say through increased wall roughness, then the convective coefficient will also increase.

### Turbulent Flow

We could develop a turbulent heat transfer correlation in a manner similar to the von Karman analysis. It is probably easier, having developed the Reynolds analogy, to follow that course. The local fluid friction factor,  $C_f$ , associated with turbulent flow over a flat plate is given as:

$$C_f = 0.0592/Re_x^{0.2}$$

Substitute into the Reynolds analogy:

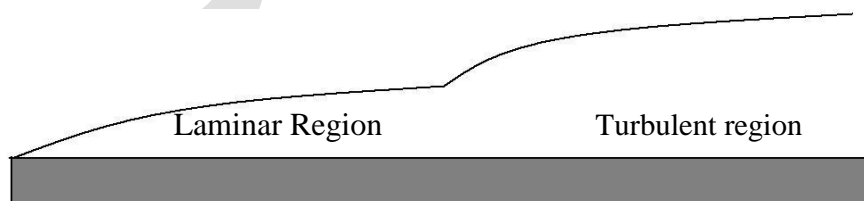
$$(0.0592/Re_x^{0.2})/2 = Nu_x/Re_x Pr^{1/3}$$

Rearrange to find

$$Nu_x = 0.0296 Re_x^{0.8} Pr^{1/3}$$

**Local Correlation  
Turbulent Flow Flat Plate.**

In order to develop an average correlation, one would evaluate an integral along the plate similar to that used in a laminar flow:



$$h_L L = \int_0^L h_x dx = \int_0^{L_{crit}} h_{x, \text{laminar}} dx + \int_{L_{crit}}^L h_{x, \text{turbulent}} dx$$

Note: The critical Reynolds number for flow over a flat plate is  $5 \times 10^5$ ; the critical Reynolds number for flow through a round tube is 2000.



The result of the above integration is:

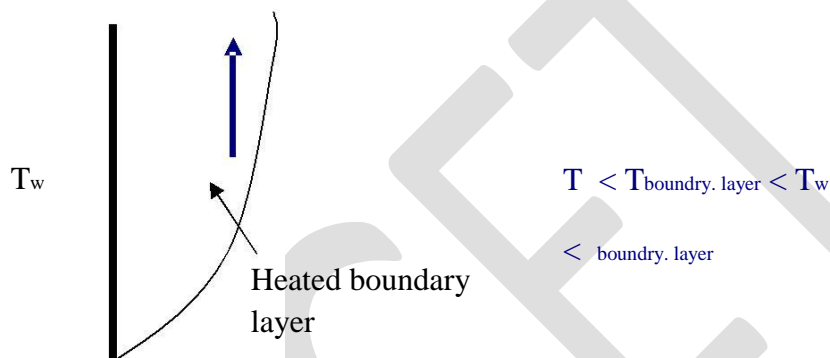
$$Nu_x = 0.037 (Re_x^{0.8} - 871) Pr^{1/3}$$

Note: Fluid properties should be evaluated at the average temperature in the boundary layer, i.e. at an average between the wall and free stream temperature.

$$T_{prop} = 0.5 (T_{wall} + T)$$

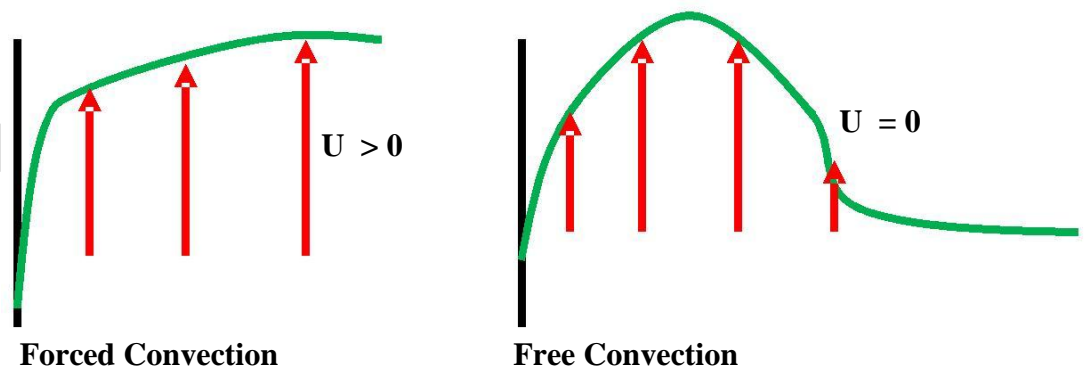
### Free convection

Free convection is sometimes defined as a convective process in which fluid motion is caused by buoyancy effects.



### Velocity Profiles

Compare the velocity profiles for forced and natural convection shown below:



### Coefficient of Volumetric Expansion

The thermodynamic property which describes the change in density leading to buoyancy in the Coefficient of Volumetric Expansion, .

## Evaluation of

Liquids and Solids: is a thermodynamic property and should be found from Property Tables. Values of are found for a number of engineering fluids in Tables given in Handbooks and Text Books.

Ideal Gases: We may develop a general expression for for an ideal gas from the ideal gas law:

$$P = R T$$

Then,

$$= P/R T$$

Differentiating while holding P constant:

$$\frac{d}{dT} \bigg|_{P \text{ Const.}} \frac{P}{R T^2} = \frac{R T}{R T^2} - \frac{P}{T^3}$$

## Grashof Number

Because U is always zero, the Reynolds number,  $[U D]/\nu$ , is also zero and is no longer suitable to describe the flow in the system. Instead, we introduce a new parameter for natural convection, the Grashof Number. Here we will be most concerned with flow across a vertical surface, so that we use the vertical distance, z or L, as the characteristic length.

Just as we have looked at the Reynolds number for a physical meaning, we may consider the Grashof number:

## Free Convection Heat Transfer Correlations

The standard form for free, or natural, convection correlations will appear much like those for forced convection except that (1) the Reynolds number is replaced with a Grashof number and (2) the exponent on Prandtl number is not generally 1/3 (The von Karman boundary layer analysis from which we developed the 1/3 exponent was for forced convection flows):

$$Nu_x = C Gr_x^m Pr^n$$

Local Correlation

$$Nu_L = C Gr_L^m Pr^n$$

Average Correlation

Quite often experimentalists find that the exponent on the Grashof and Prandtl numbers are equal so that the general correlations may be written in the form:

$$Nu_x = C [Gr_x Pr]^m$$

Local Correlation

$$Nu_L = C [Gr_L Pr]^m$$

Average Correlation

This leads to the introduction of the new, dimensionless parameter, the Rayleigh number, Ra:

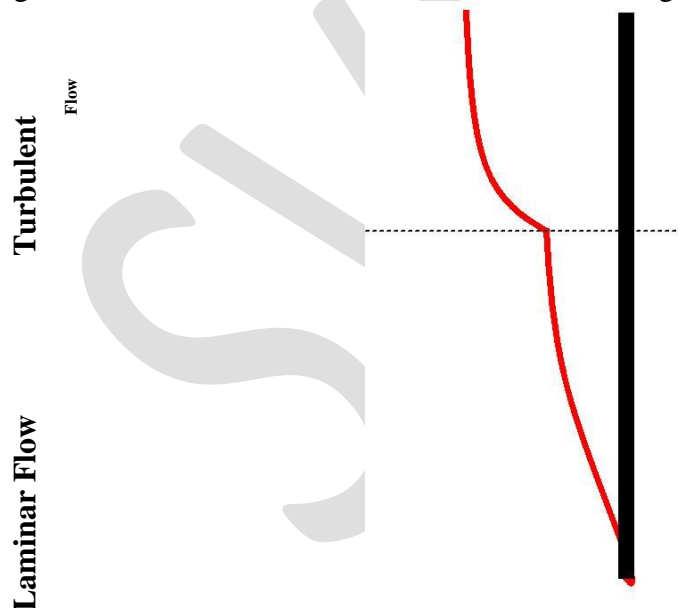
$$Ra_x = Gr_x Pr$$

$$Ra_L = Gr_L Pr$$

So that the general correlation for free convection becomes:

### Laminar to Turbulent Transition

Just as for forced convection, a boundary layer will form for free convection. The boundary layer, which acts as a thermal resistance, will be relatively thin toward the leading edge of the surface resulting in a relatively high convection coefficient. At a Rayleigh number of about  $10^9$  the flow over a flat plate will become transitional and finally become turbulent. The increased turbulence inside the boundary layer will enhance heat transfer leading to relative high convection coefficients because of better mixing.



$Ra < 10^9$  Laminar flow. [Vertical Flat Plate]

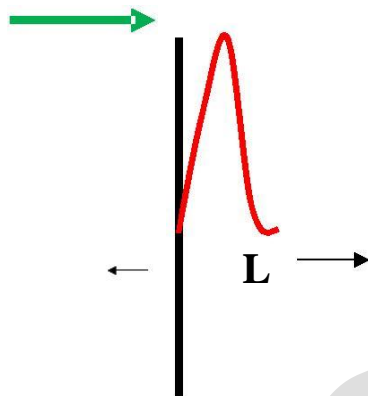
$Ra > 10^9$  Turbulent flow. [Vertical Flat Plate]

Generally the characteristic length used in the correlation relates to the distance over which the boundary layer is allowed to grow. In the case of a vertical flat plate this will be  $x$  or  $L$ , in

the case of a vertical cylinder this will also be  $x$  or  $L$ ; in the case of a horizontal cylinder, the length will be  $d$ .

### Critical Rayleigh Number

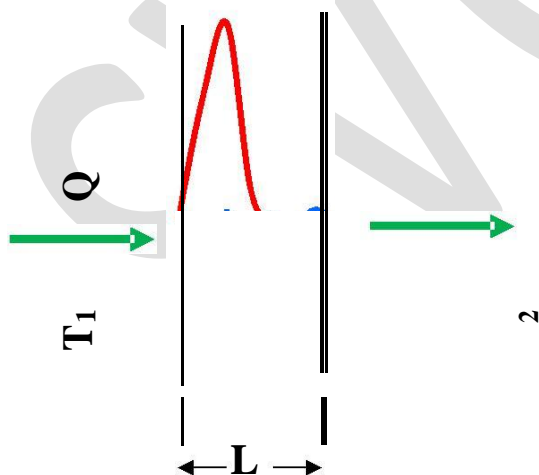
Consider the flow between two surfaces, each at different temperatures. Under developed flow conditions, the interstitial fluid will reach a temperature between the temperatures of the two surfaces and will develop free convection flow patterns. The fluid will be heated by one surface, resulting in an upward buoyant flow, and will be cooled by the other, resulting in a downward flow.



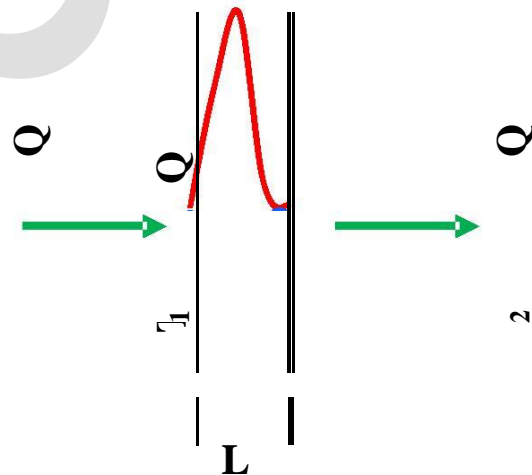
Note that for enclosures it is customary to develop correlations which describe the overall (both heated and cooled surfaces) within a single correlation.

### Free Convection Inside an Enclosure (boundary layer limit)

If the surfaces are placed closer together, the flow patterns will begin to interfere:



### Free Convection Inside an Enclosure With Partial Flow Interference



### Free Convection Inside an Enclosure With Complete Flow Interference (Channel flow limit)

In the case of complete flow interference, the upward and downward forces will cancel, canceling circulation forces. This case would be treated as a pure convection problem since no bulk transport occurs.

The transition in enclosures from convection heat transfer to conduction heat transfer occurs at what is termed the “**Critical Rayleigh Number**”. Note that this terminology is in clear contrast to forced convection where the critical Reynolds number refers to the transition from laminar to turbulent flow.

$$Ra_{crit} = 1000 \quad (\text{Enclosures With Horizontal Heat Flow})$$

$$Ra_{crit} = 1728 \quad (\text{Enclosures With Vertical Heat Flow})$$

The existence of a Critical Rayleigh number suggests that there are now three flow regimes:

(1) No flow, (2) Laminar Flow and (3) Turbulent Flow. In all enclosure problems the Rayleigh number will be calculated to determine the proper flow regime before a correlation is chosen.

# HEAT EXCHANGERS

## What are heat exchangers?

Heat exchangers are devices used to transfer heat energy from one fluid to another. Typical heat exchangers experienced by us in our daily lives include condensers and evaporators used in air conditioning units and refrigerators. Boilers and condensers in thermal power plants are examples of large industrial heat exchangers. There are heat exchangers in our automobiles in the form of radiators and oil coolers. Heat exchangers are also abundant in chemical and process industries.

There is a wide variety of heat exchangers for diverse kinds of uses, hence the construction also would differ widely. However, in spite of the variety, most heat exchangers can be classified into some common types based on some fundamental design concepts. We will consider only the more common types here for discussing some analysis and design methodologies.

## Heat Transfer Considerations

The energy flow between hot and cold streams, with hot stream in the bigger diameter tube, is as shown in Figure 7.1. Heat transfer mode is by convection on the inside as well as outside of the inner tube and by conduction across the tube. Since the heat transfer occurs across the smaller tube, it is this internal surface which controls the heat transfer process. By convention, it is the outer surface, termed  $A_o$ , of this central tube which is referred to in describing heat exchanger area. Applying the principles of thermal resistance,

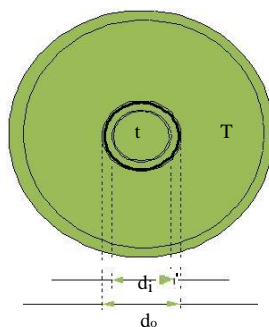


Figure 7.1: End view of a tubular heat exchanger

$$R = \frac{1}{h_o A_o} + \frac{\frac{r_o}{k} \ln \frac{r_o}{r_i}}{2 kl} + \frac{1}{h_i A_i}$$

If we define overall the heat transfer coefficient,  $U_c$ , as:

$$U_c = \frac{1}{R A_o}$$

Substituting the value of the thermal resistance  $R$  yields:

$$\frac{1}{U_c} = \frac{1}{h_o} + \frac{\frac{r_o}{k} \ln \frac{r_o}{r_i}}{k} + \frac{A_o}{h_i A_i}$$

Standard convective correlations are available in text books and handbooks for the convective coefficients,  $h_o$  and  $h_i$ . The thermal conductivity,  $k$ , corresponds to that for the material of the internal tube. To evaluate the thermal resistances, geometrical quantities (areas and radii) are determined from the internal tube dimensions available.

## Fouling

Material deposits on the surfaces of the heat exchanger tubes may add more thermal resistances to heat transfer. Such deposits, which are detrimental to the heat exchange process, are known as fouling. Fouling can be caused by a variety of reasons and may significantly affect heat exchanger performance. With the addition of fouling resistance, the overall heat transfer coefficient,  $U_c$ , may be modified as:

Fouling can be caused by the following sources:

- 1) *Scaling* is the most common form of fouling and is associated with inverse solubility salts. Examples of such salts are  $\text{CaCO}_3$ ,  $\text{CaSO}_4$ ,  $\text{Ca}_3(\text{PO}_4)_2$ ,  $\text{CaSiO}_3$ ,  $\text{Ca}(\text{OH})_2$ ,  $\text{Mg}(\text{OH})_2$ ,  $\text{MgSiO}_3$ ,  $\text{Na}_2\text{SO}_4$ ,  $\text{LiSO}_4$ , and  $\text{Li}_2\text{CO}_3$ .
- 2) *Corrosion fouling* is caused by chemical reaction of some fluid constituents with the heat exchanger tube material.
- 3) *Chemical reaction fouling* involves chemical reactions in the process stream which results in deposition of material on the heat exchanger tubes. This commonly occurs in food processing industries.
- 4) *Freezing fouling* occurs when a portion of the hot stream is cooled to near the freezing point for one of its components. This commonly occurs in refineries where paraffin frequently solidifies from petroleum products at various stages in the refining process, obstructing both flow and heat transfer.
- 5) *Biological fouling* is common where untreated water from natural resources such as rivers and lakes is used as a coolant. Biological microorganisms such as algae or other microbes can grow inside the heat exchanger and hinder heat transfer.
- 6) *Particulate fouling* results from the presence of microscale sized particles in solution. When such particles accumulate on a heat exchanger surface they sometimes fuse and harden. Like scale these deposits are difficult to remove.



## Basic Heat Exchanger Flow Arrangements

Two basic flow arrangements are as shown in Figure 7.2. Parallel and counter flow provide alternative arrangements for certain specialized applications. In parallel flow both the hot and cold streams enter the heat exchanger at the same end and travel to the opposite end in parallel streams. Energy is transferred along the length from the hot to the cold fluid so the outlet temperatures asymptotically approach each other. In a counter flow arrangement, the two streams enter at opposite ends of the heat exchanger and flow in parallel but opposite directions. Temperatures within the two streams tend to approach one another in a nearly linearly fashion resulting in a much more uniform heating pattern. Shown below the heat exchangers are representations of the axial temperature profiles for each. Parallel flow results in rapid initial rates of heat exchange near the entrance, but heat transfer rates rapidly decrease as the temperatures of the two streams approach one another. This leads to higher exergy loss during heat exchange. Counter flow provides for relatively uniform temperature differences and, consequently, lead toward relatively uniform heat rates throughout the length of the unit.

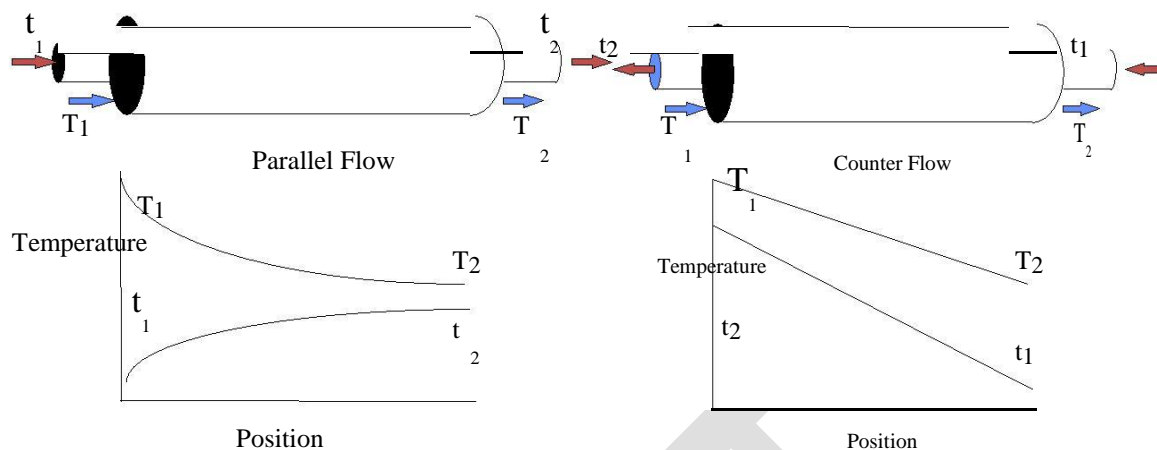


Fig. 7.2 Basic Flow Arrangements for Tubular Heat Exchangers.

### Log Mean Temperature Differences

Heat flows between the hot and cold streams due to the temperature difference across the tube acting as a driving force. As seen in the Figure 7.3, the temperature difference will vary along the length of the HX, and this must be taken into account in the analysis.

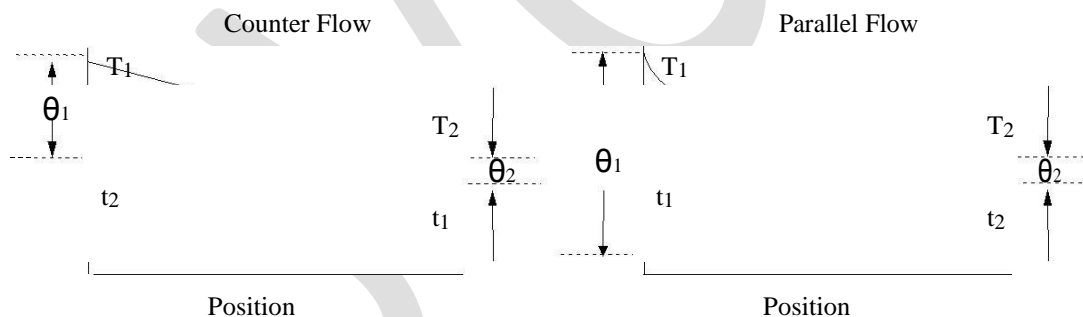


Fig. 7.3 Temperature Differences Between Hot and Cold Process Streams

From the heat exchanger equations shown earlier, it can be shown that the integrated average temperature difference for either parallel or counter flow may be written as:

The effective temperature difference calculated from this equation is known as the log mean temperature difference, frequently abbreviated as LMTD, based on the type of mathematical average that it describes. While the equation applies

to either parallel or counter flow, it can be shown that  $\epsilon_{eff}$  will always be greater in the counter flow arrangement.

Another interesting observation from the above Figure is that counter flow is more appropriate for maximum energy recovery. In a number of industrial applications there will be considerable energy available within a hot waste stream which may be recovered before the stream is discharged. This is done by recovering energy into a fresh cold stream. Note in the Figures shown above that the hot stream may be cooled to  $t_1$  for counter flow, but may only be cooled to  $t_2$  for parallel flow. Counter flow allows for a greater degree of energy recovery. Similar arguments may be made to show the advantage of counter flow for energy recovery from refrigerated cold streams.

### Applications for Counter and Parallel Flows

We have seen two advantages for counter flow, (a) larger effective LMTD and (b) greater potential energy recovery. The advantage of the larger LMTD, as seen from the heat exchanger equation, is that a larger LMTD permits a smaller heat exchanger area,  $A_o$ , for a given heat transfer,  $Q$ . This would normally be expected to result in smaller, less expensive equipment for a given application.

Sometimes, however, parallel flows are desirable (a) where the high initial heating rate may be used to advantage and (b) where it is required the temperatures developed at the tube walls are moderate. In heating very viscous fluids, parallel flow provides for rapid initial heating and consequent decrease in fluid viscosity and reduction in pumping requirement. In applications where moderation of tube wall temperatures is required, parallel flow results in cooler walls. This is especially beneficial in cases where the tubes are sensitive to fouling effects which are aggravated by high temperature.

### Multipass Flow Arrangements

In order to increase the surface area for convection relative to the fluid volume, it is common to design for multiple tubes within a single heat exchanger. With multiple tubes it is possible to arrange to flow so that one region will be in parallel and another portion in counter flow. An arrangement where the tube side fluid passes through once in parallel and once in counter flow is shown in the Figure 7.4. Normal terminology would refer to this arrangement as a 1-2 pass heat exchanger, indicating that the shell side fluid passes through the unit once, the tube side twice. By convention the number of shell side passes is

always listed first.

The primary reason for using multipass designs is to increase the average tube side fluid velocity in a given arrangement. In a two pass arrangement the fluid flows through only half the tubes and any one point, so that the Reynold's number is effectively doubled. Increasing the Reynolds's number results in increased turbulence, increased Nusselt numbers and, finally, in increased convection coefficients. Even though the parallel portion of the flow results in a lower effective T, the increase in overall heat transfer coefficient will frequently compensate so that the overall heat exchanger size will be smaller for a specific service. The improvement achievable with multipass heat exchangers is substantially large. Accordingly, it is a more accepted practice in modern industries compared to conventional true parallel or counter flow designs.

The LMTD formulas developed earlier are no longer adequate for multipass heat exchangers. Normal practice is to calculate the LMTD for counter flow,  $LMTD_{cf}$ , and to apply a correction factor,  $F_T$ , such that

$$_{eff} F_T LMTD_{CF}$$

The correction factors,  $F_T$ , can be found theoretically and presented in analytical form. The equation given below has been shown to be accurate for any arrangement having 2, 4, 6, ....., 2n tube passes per shell pass to within 2%.

$$F_T = \frac{\sqrt{R^2 + 1} \ln \frac{1 + P}{1 - R P}}{R \ln 2 \frac{P R + 1}{2 P R + 1} \frac{\sqrt{R^2 + 1}}{\sqrt{R^2 + 1}}}$$

where the capacity ratio, R, is defined as:

$$R = \frac{T_1 - T_2}{t_2 - t_1}$$

The effectiveness may be given by the equation:

$$\frac{1 - X^{1/N_{shell}}}{R X^{1/N_{shell}}}$$

provided that  $R \neq 1$ . In the case that  $R=1$ , the effectiveness is given by:

$$\frac{P_{N_{shell}} - P_o}{P_o} \frac{P_o}{P_{N_{shell}}} = 1$$

where

$$P_o = \frac{t_2 - t_1}{T_1 - t_1}$$

and

$$X = \frac{P_o}{P_{N_{shell}}} \frac{R-1}{P_o}$$

### Effectiveness-NTU Method:

Quite often, heat exchanger analysts are faced with the situation that only the inlet temperatures are known and the heat transfer characteristics ( $UA$  value) are known, but the outlet temperatures have to be calculated. Clearly, LMTH method will not be applicable here. In this regard, an alternative method known as the  $\epsilon$ -NTU method is used.

Before we introduce this method, let us ask ourselves following question: Define effectiveness : How will existing Heat Exchange perform for given inlet conditions ?

, where  $Q_{max}$  is for an infinitely long H.Ex. The effectiveness,  $\epsilon$ , is the ratio of the energy recovered

in a HX to that recoverable in an ideal HX.

$$\frac{Q_{actual}}{Q_{max}}$$

$$Q_{max}$$

$$\text{One fluid } T_{max} - T_{h,in} \quad T_{c,in}$$

$$\text{and since } Q = m C_A (T_A - T_B) = m C_B (T_B - T_A)$$

then only the fluid with lesser of  $C_A, C_B$  heat capacity rate can have  $T_{max}$

$$\text{i.e. } Q_{max} = C_{min} (T_{max} - T_{c,in}) \quad \text{and} \quad \frac{Q}{C_{min} (T_{h,in} - T_{c,in})}$$

$$\text{or, } Q = C_{min} (T_{h,in} - T_{c,in})$$

We want expression for which does not contain outlet T's.

Substitute back into  $Q = UA(LMTD)$  .....  
- UA

$$\frac{1 - \exp\left(-\frac{C_{\min}}{C_{\max}}\right)}{1 - \frac{C_{\min}}{C_{\max}}} \exp\left(-\frac{UA}{C_{\min}}\right) = \frac{1 - \exp\left(-\frac{C_{\min}}{C_{\max}}\right)}{1 - \frac{C_{\min}}{C_{\max}}}$$

SVCE

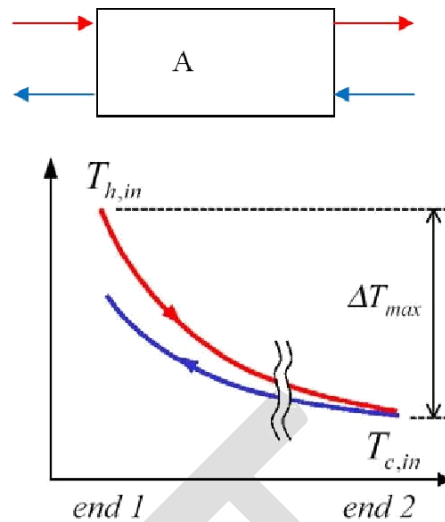


Fig. 7.5 Calculation of effectiveness-NTU

NTU ,

$$\frac{C_{min}}{C_{max}}$$

and No. of transfer units (size of HEx.)  $NTU = \frac{UA}{C_{min}}$

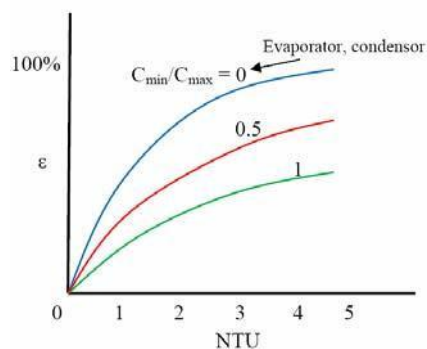
### Charts for each Configuration

#### Procedure:

Determine  $C_{max}$ ,  $C_{min}/C_{max}$

Get  $UA/C_{min}$ , from chart

$$Q = C_{min} (T_{h,in} - T_{c,in})$$



$$NTU_{\max} = \frac{UA}{C_{\min}} = A \frac{NTU_{\max} C_{\min}}{U}$$

- $NTU_{\max}$  can be obtained from figures in textbooks/handbooks First, however, we must determine which fluid has  $C_{\min}$ .



# BOILING AND CONDENSATION

## 1 Boiling: General considerations

Boiling is associated with transformation of liquid to vapor at a solid/liquid interface due to convection heat transfer from the solid.

Agitation of fluid by vapor bubbles provides for large convection coefficients and hence large heat fluxes at low-to-moderate surface-to-fluid temperature differences

Special form of Newton's law of cooling:

$$q_s = h(T_s - T_{sat}) = h T_e$$

where  $T_{sat}$  is the saturation temperature of the liquid, and  $T_e = T_s - T_{sat}$  is the excess temperature.

## 4) Special cases

Pool Boiling: Liquid motion is due to natural convection and bubble-induced mixing.

Forced Convection Boiling: Fluid motion is induced by external means, as well as by bubble-induced mixing.

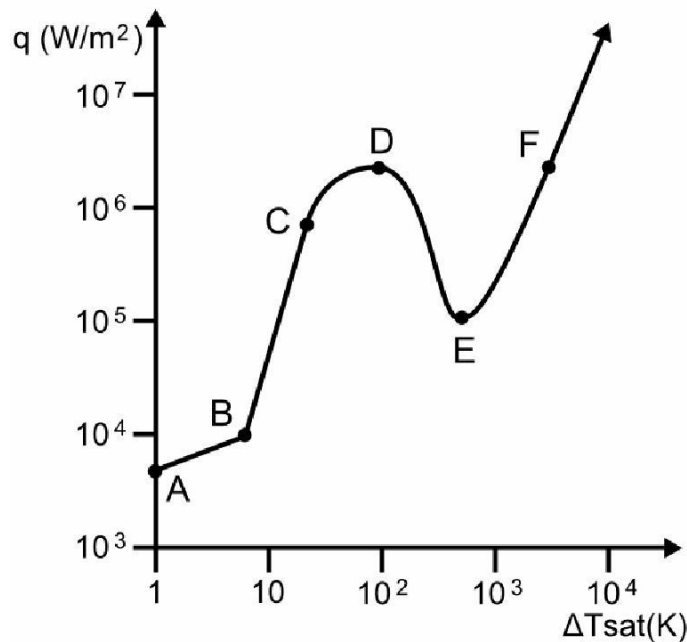
Saturated Boiling: Liquid temperature is slightly larger than saturation temperature

Subcooled Boiling: Liquid temperature is less than saturation temperature

## 5) The boiling curve

The boiling curve reveals range of conditions associated with saturated pool boiling on a  $q_s$  vs.  $T_e$  plot.

## Water at Atmospheric Pressure



### Pool boiling regimes:

A-B: Pure convection with liquid rising to surface for evaporation  
 B-C: Nucleate boiling with bubbles condensing in liquid  
 C-D: Nucleate boiling with bubbles rising to surface  
 D: Peak temperature  
 D-E: Partial nucleate boiling and unstable film boiling  
 E: Film boiling is stabilized  
 E-F: Radiation becomes a dominant mechanism for heat transfer

### Free Convection Boiling ( $T_e < 5^\circ\text{C}$ )

- Little vapor formation.
- Liquid motion is due principally to single-phase natural convection.

### Onset of Nucleate Boiling – ONB ( $T_e \approx 5^\circ\text{C}$ )

### Nucleate boiling ( $5^\circ\text{C} < T_e < 30^\circ\text{C}$ )

- Isolated Vapor Bubbles ( $5^\circ\text{C} < T_e < 10^\circ\text{C}$ )

Liquid motion is strongly influenced by nucleation of bubbles at the surface.

$h$  and  $q_s$  rise sharply with increasing  $T_e$

Heat transfer is principally due to contact of liquid with the surface (single-phase convection) and not to vaporization

- Jets and Columns ( $10^{\circ}\text{C} < T_e < 30^{\circ}\text{C}$ )

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Increasing number of nucleation sites causes bubble interactions and coalescence into jets and slugs.

Liquid/surface contact is impaired.

$q_s$  continues to increase with  $T_e$  while  $h$  begins to decrease

Critical Heat Flux - CHF, ( $T_e = 30^\circ\text{C}$ )

- Maximum attainable heat flux in nucleate boiling.

pressure.

Potential Burnout for Power-Controlled Heating

- An increase in  $q_s$  beyond  $q_{\max}$  causes the surface to be blanketed by vapor and its temperature to spontaneously achieve a value that can exceed its melting point
- If the surface survives the temperature shock, conditions are characterized by film boiling

Film Boiling

- Heat transfer is by conduction and radiation across the vapor blanket
- A reduction in  $q_s$  follows the cooling curve continuously to the Leidenfrost point corresponding to the minimum heat flux  $q_{\min}$  for film boiling.
- A reduction in  $q_s$  below  $q_{\min}$  causes an abrupt reduction in surface temperature to the nucleate boiling regime

Transition Boiling for Temperature-Controlled Heating

- Characterised by continuous decay of  $q_s$  (from  $q_{\max}$  to  $q_{\min}$ )

with increasing  $T_e$

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- Surface conditions oscillate between nucleate and film boiling, but portion of surface experiencing film boiling increases with  $T_e$
- Also termed unstable or partial film boiling.

## 8.4 Pool boiling correlations

### Nucleate Boiling

- Rohsenow Correlation, clean surfaces only,  $\pm 100\%$  errors

$$q_c = h_{fg} \left[ \frac{g}{l_v} \right]^{1/2} \left[ \frac{C_{p,l} T_e}{C_{s,f} h_{fg} Pr_l} \right]^{1/4}$$

$C_{s,f}$ , nSurface/Fluid Combination

Critical heat flux:

$$q_{max} = 0.149 h_{fg} \left[ \frac{g}{l_v} \right]^{1/4}$$

### Film Boiling

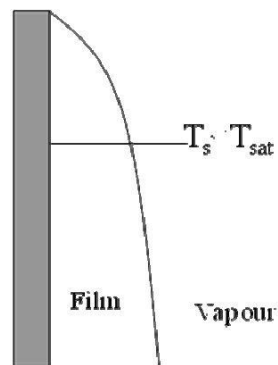
$$Nu_D = \frac{h_{conv} D}{k_v} = C \left[ \frac{g}{l_v} \right]^{1/4} \left[ \frac{h_{fg} D}{k_v (T_s - T_{sat})} \right]^{1/4}$$

Geometry	C
Cylinder(Hor.)	0.62
Sphere	0.67

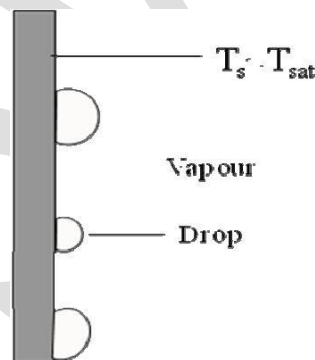
## 8.5 Condensation: General considerations

- Condensation occurs when the temperature of a vapour is reduced below its saturation temperature
- Condensation heat transfer

### *Film condensation*

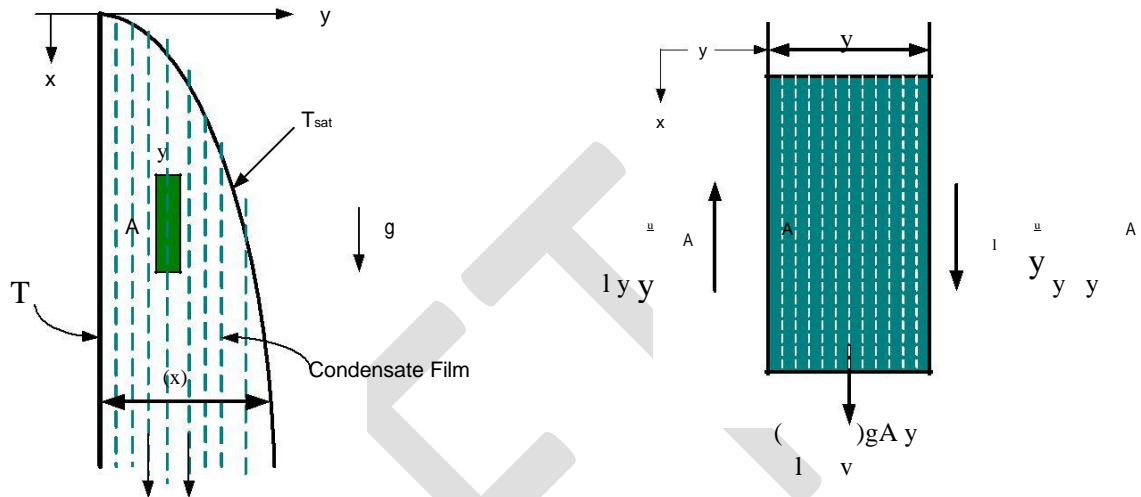


### *Dropwise condensation*



- Heat transfer rates in dropwise condensation *may be as much as 10 times higher* than in film condensation

## 8.6 Laminar film condensation on a vertical wall



$$q = h_L A (T_{sat} - T_w)$$

$$\frac{q}{m} = \frac{\bar{h}_L A (T_{sat} - T_w)}{h_{fg}}$$